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THE OPTIMAL PRODUCERS' ADJUSTMENT TRAJECTORY[2]

1. INTRODUCTION

Let us consider the private ownership economy (see e.g. Debreu, 1959; Mas-Colell et al., 1995). Let us also suppose that for some reasons (for instance new regulations, new technologies etc.) producers have to modify their technologies. It can result in a mild evolution within the production sector which does not, however, disturb the equilibrium. That evolution indicates at the time the current system of private ownership economies. Hence, this survey can be also viewed as an attempt to model changes of the producers' sphere of the private ownership economy (compare to Radner, 1972 or Magill, Quinzii, 2002).

This paper is a continuation of the research originated in Lipieta (2010) where a model of the private ownership economy with complementary commodities was presented. Later, changes of the production system of the private ownership economy were studied in a generalized form of the economy with complementary commodities, the so called economy with the reduced consumption sphere (see Lipieta, 2012).

The mapping describing changes introduced by producers will be called the producers' trajectory. Keeping equilibrium (where applicable) and minimization of the distance between the initial and final production plans are considered as the main criterion of the choice of the producers' trajectory. Hence, projections defined in commodity-price space $\mathbb{R}^{\ell}$ with maximum norm are used for modeling changes in the production sphere.

The paper consists of four parts. The second section presents the construction of the private ownership economy. The third part deals with the description of such a modification of the production sphere of the private ownership economy that does not disturb equilibrium. The fourth part presents the characterization of the best trajectory of changes of the economy under study with respect to the criterion of the distance minimization.

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2. THE MODEL

The private ownership economy defined in Debreu (1959) is studied in the form of a multi-range relational system which includes the combination of production and consumption systems (see Lipieta, 2010; 2013). The linear space $\mathbb{R}^\ell$ ($\ell \in \{1,2,\dots\}$) with the scalar product

$$(x \circ y) = (x_1, \dots, x_\ell) \circ (y_1, \dots, y_\ell) = \sum_{k=1}^{\ell} x_k \cdot y_k,$$

$x, y \in \mathbb{R}^\ell$, is the ℓ-dimensional commodity-price space. Suppose that two groups of agents viz. producers and consumers, operate in $\mathbb{R}^\ell$. Let $n \in \{1,2,\dots\}$ and

– $B = \{b_1, \dots, b_n\}$ be a finite set of producers,
– $y: B \ni b \to Y^b \subset \mathbb{R}^\ell$ be the correspondence of production sets, which to every producer b assigns a nonempty production set $y(b) = Y^b \subset \mathbb{R}^\ell$ of the producer's feasible production plans,
– $p \in \mathbb{R}^\ell$ be a price vector.

Definition 2.1. A two-range relational system $P_q = (B, \mathbb{R}^\ell; y, p)$, is called the quasi--production system.

Definition 2.2. If $P_q = (B, \mathbb{R}^\ell; y, p)$ is the quasi-production system, where

$$\forall\, b \in B \quad \eta^b(p) \overset{\text{def}}{=} \{y^{b*} \in y(b): p \circ y^{b*} = \max\{p \circ y^b: y^b \in y(b)\}\} \neq \emptyset,$$

then

– $\eta: B \ni b \to \eta^b(p) \subset \mathbb{R}^\ell$ is called the correspondence of supply at price system p,
– $\pi: B \ni b \to \pi(b) = p \circ y^{b*} \in \mathbb{R}$, where $y^{b*} \in \eta^b(p)$, is called the maximal profit function at price system p,
– the quasi-production system P_q is called the production system and denoted by

$$P_q = P = (B, \mathbb{R}^\ell; y, p, \eta, \pi).$$

The set $\eta^b(p)$ is called the set of optimal plans of producer b at given price vector p.

In the quasi-production system, the aim of producers is not specified in contrast to the production system, where the producers aim at profits maximization at given prices and technologies. In quasi-production system $P_q = (B, \mathbb{R}^\ell; y, p)$, the profit function of a producer b at price vector p, is of the form:

$$Y^b \ni y^b \to p \circ y^b \in \mathbb{R}.$$

Let $\hat{y}^b \in y(b)$ denote the plan realized by producer $b \in B$. If $\hat{y}^b$ is the optimal plan of producer b at given price vector p, then it will be noted by y^{b*} ($\hat{y}^b = y^{b*}$) and

$$p \circ \hat{y}^b = p \circ y^{b*} = \max\{p \circ y^b : y^b \in y(b)\}.$$

Now, the consumption sphere is defined. Let $m \in \{1, 2, \ldots\}$ and

- $A = \{a_1, \ldots, a_m\}$ be a finite set of consumers,
- $\Xi \subset \mathbb{R}^\ell \times \mathbb{R}^\ell$ be the family of all preference relations in $\mathbb{R}^\ell$,
- $\chi: A \ni a \to \chi(a) = X^a \subset \mathbb{R}^\ell$ be a correspondence of consumptions sets,
- $e: A \ni a \to e(a) \in \mathbb{R}^\ell$ be an initial endowment mapping,
- $\varepsilon \subset A \times (\mathbb{R}^\ell \times \mathbb{R}^\ell)$ be a correspondence, which assigns a preference relation $\preccurlyeq^a$ to every consumer $a \in A$ from set Ξ restricted to set $\chi(a) \times \chi(a)$,
- $p \in \mathbb{R}^\ell$ be a price vector.

Definition 2.3. The three-range relational system $C_q = (A, \mathbb{R}^\ell, \Xi;\ \chi, e, \varepsilon, p)$ is called the quasi-consumption system.

However, we assume that if consumer $a \in A$ has a possibility to maximize his preference relation on the budget set, then he uses that opportunity. It should be noted that the expenditures of every consumer $a \in A$ in quasi-consumption system C_q cannot be greater than the value

$$w(a) = p \circ e(a). \tag{1}$$

Vector (1) is called the wealth of consumer a.

Let $C_q = (A, \mathbb{R}^\ell, \Xi; \chi, e, \varepsilon, p)$ be the quasi-consumption system.

Definition 2.4. If at the given price vector $p \in \mathbb{R}^\ell$, for every $a \in A$

$$\beta(a) = \beta^a(p) = \{x \in \chi(a): p \circ x \leq w(a)\} \neq \emptyset. \tag{2}$$

$$\varphi(a) = \varphi^a(p) = \{x^{a*} \in \beta^a(p): \forall x^a \in\ \beta^a(p)\ x^a \preccurlyeq^a x^{a*}, \preccurlyeq^a \in \Xi\} \neq \emptyset, \tag{3}$$

then

- $\beta: A \ni a \to \beta^a(p) \subset \mathbb{R}^\ell$ is the correspondence of budget sets at price system p, which to every consumer $a \in A$ assigns his set of budget constrains $\beta^a(p) \subset \chi(a)$ at price system p and initial endowment $e(a)$,
- $\varphi: A \ni a \to \varphi^a(p) \subset \mathbb{R}^\ell$ is the demand correspondence at price system p, which to every consumer $a \in A$ assigns the consumption plans maximizing his preference on the budget set $\beta^a(p)$,

– the quasi-consumption system C_q is called the consumption system and denoted by

$$C_q = C = (A, \mathbb{R}^\ell, \Xi;\ \chi, e, \varepsilon, p, \beta, \varphi).$$

Let $p \in \mathbb{R}^\ell$ be a price vector. The following definition may be assumed on the basis of the above:

Definition 2.5. The relational system $E_q = \left(\mathbb{R}^\ell, P_q, C_q, \theta, \omega\right)$, where
– $P_q = (B, \mathbb{R}^\ell; y, p)$ is the quasi-production system,
– the mapping $\theta: A \times B \to [0,1]$ satisfies,

$$\forall b \in B\ \textstyle\sum_{a \in A} \theta(a, b) = 1, \tag{4}$$

– $C_q = (A, \mathbb{R}^\ell, \Xi;\ \chi, e, \varepsilon, p)$ is the quasi-consumption system in which

$$w(a) = p \circ e(a) + \textstyle\sum_{b \in B} \theta(a, b) \cdot p \circ \hat{y}^b, \tag{5}$$

– $\sum_{a \in A} e(a) = \omega \in \mathbb{R}^\ell$ (6)

is called the private ownership economy.

If P_q is production system ($P_q = P$) and C_q is consumption system ($C_q = C$), then private ownership economy E_q will be called the Debreu economy and denoted by

$$E_p = E_q = (\mathbb{R}^\ell, P, C, \theta, \omega).$$

Number $\theta(a, b)$ indicates that part of the profit of producer b which is owned by consumer a. The private ownership economy E_q operates as follows. Let a price vector $p \in \mathbb{R}^\ell$ be given. Every producer b realizes a production plan $\hat{y}^b \in y(b)$. The profit of each producer b, by realization of the plan $\hat{y}^b$, is divided among all consumers according to function θ (see (4)). So, the expenditures of every consumer $a (a \in A)$ cannot be greater than value $w(a)$ (see (5)). If $\beta^a(p) \neq \emptyset$ (see (2)) and $\varphi^a(p) \neq \emptyset$ (see (3)), then consumer a chooses his consumption plan $\hat{x}^a = x^{a*} \in \varphi^a(p) \subset \chi(a)$ maximizing his preference on budget set $\beta^a(p)$. If $\beta^a(p) \neq \emptyset$ and $\varphi^a(p) = \emptyset$, then consumer a chooses his consumption plan $\hat{x}^a \in \beta^a(p)$, due to his own criterion. If $\beta^a(p) = \emptyset$, then we assume that $\hat{x}^a = 0 \in \mathbb{R}^\ell$. If

$$\textstyle\sum_{a \in A} \hat{x}^a - \sum_{b \in B} \hat{y}^b = \omega, \tag{7}$$

then it is said that there is quasi-equilibrium in economy E_q and vector p is called the quasi-equilibrium price vector in that economy. Consequently, the sequence

$$((\hat{x}^a)_{a \in A}, (\hat{y}^b)_{b \in B}, p) \stackrel{\text{def}}{=} (\hat{x}^{a_1}, \dots, \hat{x}^{a_m}, \hat{y}^{b_1}, \dots, \hat{y}^{b_n}, p) \in (\mathbb{R}^\ell)^{m+n+1} \tag{8}$$

is called the state of quasi-equilibrium in economy E_q. If economy E_q is the Debreu economy ($E_q = E_p$, see def. 2.5), then the sequence

$$((x^{a*})_{a\in A}, (y^{b*})_{b\in B}, p) \overset{\text{def}}{=} (x^{a_1*}, \dots, x^{a_m*},\ y^{b_1*}, \dots, y^{b_n*}, p) \in (\mathbb{R}^{\ell})^{m+n+1}, \quad (9)$$

that satisfies

$$\sum_{a\in A} x^{a*} - \sum_{b\in B} y^{b*} = \omega, \quad (10)$$

is the state of equilibrium in Debreu economy E_p. If condition (10) is satisfied, then it is said that there is equilibrium in economy E_p and vector p is called the equilibrium price vector in that economy.

3. SYSTEM OF PRIVATE OWNERSHIP ECONOMIES

At first, we recall some properties of subspaces of $\mathbb{R}^{\ell}$ that will be in use later. Let $V \subset \mathbb{R}^{\ell}$ be a linear subspace of dimension $\ell - k,\ k \in \{1, \dots, \ell - 1\}$. Then there exist linearly independent vectors $g^1, \dots, g^k \in \mathbb{R}^{\ell}$ such that

$$V = \bigcap_{s=1}^{k} \ker \tilde{g}^s, \quad (11)$$

where, for $s \in \{1, 2, \dots, k\}$,

$$\tilde{g}^s \colon \mathbb{R}^{\ell} \ni (x_1, \dots, x_{\ell}) \to \sum_{l=1}^{\ell} g_l^s x_l \in \mathbb{R} \quad (12)$$

and

$$\ker \tilde{g}^s = \{x = (x_1, \dots, x_{\ell}) \in \mathbb{R}^{\ell} \colon \tilde{g}^s(x) = 0\}. \quad (13)$$

Now, we put the following definition:

Definition 3.1 (see Lipieta, 2012). The private ownership economy $E_q = (P, C, \theta, \omega)$, in which condition

$$\forall a \in A\ \ X^a \subset V \quad (14)$$

is satisfied, will be called the private ownership economy with the reduced consumption system.

The private ownership economy $E_q = (P, C, \theta, \omega)$, in which condition

$$\forall b \in B\ \ Y^b \subset V \quad (15)$$

is satisfied, will be called the private ownership economy with the reduced production system.

If there is a proper subspace V of commodity-price space $\mathbb{R}^\ell$ ($\{0\} \neq V \subset \mathbb{R}^\ell$) such that conditions (14) and (15) are both satisfied in economy E_q, then this economy will be called the private ownership economy reduced to the subspace V.

The sets satisfying condition (14) or (15) are the linear sets (see for example Moore, 2007). Hence, the economy with the reduced consumption (production) system is also called the economy with linear consumption (production) sets.
Let us notice that assumption (14) has an economic interpretation. If the consumers are not interested in the consumption of a commodity $l_0 \in \{1, \dots, \ell\}$, then the coordinate l_0 is equal 0 in every plan $x^a \in X^a$, namely

$$\forall a \in A \; x^a_{l_0} = 0.$$

Hence,

$$\forall a \in A \;\; X^a \subset \ker\tilde{g}, \tag{16}$$

where $\tilde{g}$ is of the form (12), precisely

$$\tilde{g}\colon \mathbb{R}^\ell \ni (x_1, \dots, x_\ell) \to x_{l_0} \in \mathbb{R}. \tag{17}$$

Suppose that producers' output $l_0 \in \{1, \dots, \ell\}$ is not wanted by the consumers or it is a harmful commodity. The producers, for which l_0 is the output, have to modify their plans of action and stop producing that commodity. After modification, the condition (15) will be valid, namely

$$\forall b \in B \;\; Y^b \subset \ker\tilde{g}, \tag{18}$$

with $\tilde{g}$ is of the form (17).
If there exist two commodities l_1, $l_2 \in \{1, \dots, \ell\}$ such that

$$\exists c > 0 \; \forall a \in A \; \forall x^a = (x^a_1, \dots, x^a_\ell) \in X^a \;\; x^a_{l_1} = c \cdot x^a_{l_2},$$

then commodities l_1 and l_2 are called complementary (see Lipieta, 2010). In that case, condition (16) is fulfilled with the functional of the form

$$\tilde{g}\colon \mathbb{R}^\ell \ni (x_1, \dots, x_\ell) \to x_{l_1} - c \cdot x_{l_2} \in \mathbb{R}. \tag{19}$$

Generally, if there exist numbers $c_1, c_2, \dots, c_\ell \in \mathbb{R}$ such that $\sum_{l=1}^{\ell}(c_l)^2 \neq 0$ and

$$\forall a \in A \;\; \forall x^a = (x^a_1, \dots, x^a_\ell) \in X^a \;\; \sum_{l=1}^{\ell} c_l x^a_l = 0, \tag{20}$$

we will say that the commodities for which $c_l \neq 0$ ($l \in \{1, \dots, \ell\}$) are dependent in the consumption sets. If condition (20) is satisfied, then (16) is fulfilled with functional $\tilde{g}$ of the form (12), namely

$$\tilde{g}: \mathbb{R}^\ell \ni (x_1, \dots, x_\ell) \to \sum_{l=1}^{\ell} c_l x_l \in \mathbb{R}. \tag{21}$$

If $\tilde{g}$ is of the form (21), then set ker$\tilde{g}$ is the linear subspace of $\mathbb{R}^\ell$ of dimension $\ell - 1$ (see (11)). Let us notice that if

$$\forall a \in A \;\; x^a_{l_1} = x^a_{l_2} = 0,$$

then (14) is satisfied for subspace V defined, in the meaning of condition (11), by functionals $\tilde{g}^1$ and $\tilde{g}^2$ of the form (17) for l_0 equals respectively l_1 or l_2. Hence, in the sense of condition (20) the commodities l_1, l_2 are also the complementary ones.

In many real economies, the producers are obliged to reduce the amount of pollution emitted to the atmosphere. The amount of pollution increases with the quantities of goods. Hence, saying about the dependent commodities in production sets makes sense. As in the case of consumers, if there exist real numbers $c_1, c_2, \dots, c_\ell$ such that $\sum_{l=1}^{\ell}(c_l)^2 \neq 0$ and

$$\forall b \in B \;\; \forall y^b = (y^b_1, \dots, y^b_\ell) \in Y^b \;\; \sum_{l=1}^{\ell} c_l y^b_l = 0, \tag{22}$$

we say that the commodities for which $c_l \neq 0$ ($l \in \{1, \dots, \ell\}$) are dependent in production sets.

Reducing the amount of an output l_0 in all production plans relies on decreasing in coordinate $y^b_{l_o}$ in every plan y^b of every producer b. If producers have to modify their technologies to get the desired dependency between quantities of some commodities, then they will change their plans of action to satisfy condition (15) with subspace V defined by using functionals of the form (21).

To sum up: legal requirements, new technologies, new fashions, inventions and many other reasons can contribute to the modification of production sets, to the sets satisfying (15), with subspace V of the form (11). Moreover, the rationality of producers' behavior implies that all profitable changes is production sphere are worth, in the opinion of producers, realizing.

Let us notice that the producers will not want to stop producing commodities used only in the producers' activities, namely the commodities that are outputs and inputs only for the producers. Although these commodities are not wanted by the consumers, they play an important role in the production sector.

In the next part of the paper, the procedure of such modification of the production sets will be presented that the modified producers' sets will satisfy condition (15) with a nontrivial subspace V of space $\mathbb{R}^\ell$. At first, some notations and definitions will be introduced. Let $V \subset \mathbb{R}^\ell$, $V \neq \{0\}$ be a linear subspace. Then V^T means the linear subspace orthogonal to V, namely

$$V^T \overset{\text{def}}{=} \{x \in \mathbb{R}^\ell |\, \forall v \in V: x \circ v = 0\}.$$

Fix linearly independent functionals $\tilde{g}^1, \dots, \tilde{g}^k$ of the form (12) satisfying (11). In this situation, the system of equations

$$\tilde{g}^s(x) = g^s \circ x = \delta^{sr} \quad \text{for} \quad s, r \in \{1, \dots, k\}, \tag{23}$$

where $x \in \mathbb{R}^\ell$ and

$$\delta^{sr} = \begin{cases} 1 & \text{if} \quad s = r \\ 0 & \text{if} \quad s \neq r \end{cases}$$

is Kronecker delta, has a solution. We will denote a solution of (23) by $q^1, \dots, q^k \in \mathbb{R}^\ell$. Now, we define mapping $\tilde{Q}: \mathbb{R}^\ell \times [0,1] \to \mathbb{R}^\ell$ by the rules

$$\tilde{Q}(x, t) = x - t \cdot \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s. \tag{24}$$

Notice that for every fixed $t \in [0,1]$ mapping $\tilde{Q}(\cdot, t): \mathbb{R}^\ell \to \mathbb{R}^\ell$ is the linear and continuous operator. Moreover, $\tilde{Q}(\cdot\,,1): \mathbb{R}^\ell \to \mathbb{R}^\ell$, precisely

$$\tilde{Q}(x, 1) = x - \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s \quad \text{for} \quad x \in \mathbb{R}^\ell$$

is the linear continuous projection from $\mathbb{R}^\ell$ into V, where

$$\forall x \in \mathbb{R}^\ell \;\; \tilde{Q}(x, 1) \in V \;\; \text{and} \;\; \forall v \in V \;\; \tilde{Q}(v, 1) = v$$

(see for example Cheney, 1966). From now, the projection $\tilde{Q}(\cdot\,,1)$ will be denoted by Q. Hence

$$Q(x) \overset{\text{def}}{=} \tilde{Q}(x, 1) = x - \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s \quad \text{for} \quad x \in \mathbb{R}^\ell. \tag{25}$$

It should be noted that

$$\forall v \in V \;\; \forall t \in [0,1] \;\; \tilde{Q}(v, t) = v = Q(v). \tag{26}$$

We also say that vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ determine (or define) the mappings Q, $\tilde{Q}$ and $\tilde{Q}(\cdot, t)$ for every $t \in [0,1]$. The set of all projection from $\mathbb{R}^\ell$ into subspace V will be denoted by $\mathcal{P}(\mathbb{R}^\ell, V)$. Let us recall (see Cheney, 1966) that for every $Q \in \mathcal{P}(\mathbb{R}^\ell, V)$ there exist vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ satisfying (23) such that, the projection Q is of the form (25). The above defined objects lead us to the following:

Theorem 3.2. If $p \in \mathbb{R}^\ell \backslash V^T$ then there exists continuous and linear operator $\tilde{Q}: \mathbb{R}^\ell \times [0,1] \to \mathbb{R}^\ell$ of the form (24) satisfying

$$\forall x \in \mathbb{R}^\ell \ \forall t \in [0,1] \ \ p \circ x = p \circ \tilde{Q}(x,t). \tag{27}$$

Proof. Notice that if $p \notin V^T$ then vectors $p, g^1, \dots, g^k$ are linearly independent and the system of equalities

$$\begin{cases} g^s \circ x = \delta^{sr} \\ p \circ x = 0 \end{cases} \quad s, r \in \{1, \dots, k\}. \tag{28}$$

has a solution. The solution of (28) will be also denoted by $q^1, \dots, q^k \in \mathbb{R}^\ell$. The operator $\tilde{Q}$ of the form (24), determined by vectors $q^1, \dots, q^k$, satisfies the thesis of the theorem. □

The result of the theorem 3.2 implies the following:

Theorem 3.3. Let $P = (B, \mathbb{R}^\ell; y, p, \eta, \pi)$ be a production system. There is an operator $\tilde{Q}$ of the form (24) such that for every $b \in B$ and $y^{b*} \in \eta^b(p)$, vector $\tilde{Q}(y^{b*}, t)$ maximizes, for every $t \in [0,1]$, the profit of producer b at price p, on the modified production set

$$\tilde{Q}(Y^b, t) = \{\tilde{Q}(y^b, t) \in \mathbb{R}^\ell : y^b \in Y^b\}. \tag{29}$$

Proof. If $p \notin V^T$ then the thesis of the theorem is the immediate consequence of theorem 3.2. If $p \in V^T$, then by (26) we get that every operator $\tilde{Q} \in \mathcal{P}(\mathbb{R}^\ell, V)$ of the form (24) determined by vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ calculated by (23) satisfies

$$\forall v \in V \ \forall t \in [0,1] \ \ p \circ v = p \circ \tilde{Q}(v,t) = 0 \tag{30}$$

which gives the result. □

The operator $\tilde{Q}$ by the thesis of theorem 3.3 is called the producers' adjustment trajectory.

Let $\tilde{Q}$ be an operator of form (24) and $P_q = (B, \mathbb{R}^\ell; y, p)$ be a quasi-production system. Replacing at every $t \in [0,1]$, producers' sets $Y^{b_1}, Y^{b_2}, \dots, Y^{b_n}$ in quasi-production system P_q with the sets $\tilde{Q}(Y^{b_1}, t), \tilde{Q}(Y^{b_2}, t), \dots, \tilde{Q}(Y^{b_n}, t)$, we receive also the quasi-production system. Such modified quasi-production system differs from the initial one (see def. 2.2) with correspondence of production sets. Additionally, if P_q ($P_q = P$) is the production system and $\tilde{Q}$ is the mapping by the thesis of theorem 3.3, then the modified production system is different from the initial one, also in the correspondence of supply at the given price system.

Fix $p \in \mathbb{R}^\ell$. Consider vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ satisfying (28) if $p \in \mathbb{R}^\ell \setminus V^T$ and (23) if $p \in V^T$. Let $\tilde{Q}$ be an operator of form (24) determined by vectors $q^1, \dots, q^k$ and $P_q = (B, \mathbb{R}^\ell; y, p)$ be a quasi-production system. For every $t \in [0,1]$, we have

Definition 3.4. The two-range relational system

$$P_q(q^1, \dots, q^k; t) = (B, \mathbb{R}^\ell; y_t, p)$$

where

– $y_t: B \ni b \to \tilde{Q}(Y^b, t) \subset \mathbb{R}^\ell$ is the correspondence of production sets, which assigns the image of production set Y^b to every $b \in B$ producer by mapping $\tilde{Q}(\cdot, t)$, is called the modification of system P_q, at time t, determined by vectors $q^1, \dots, q^k$.
If quasi-production system P_q is the production system $P_q = P = (B, \mathbb{R}^\ell; y, p, \eta, \pi)$, then two-range relational system

$$P(q^1, \dots, q^k; t) = (B, \mathbb{R}^\ell; y_t, p, \eta_t, \pi_t)$$

where additionally

– $\eta_t: B \ni b \to \eta_t^b(p) \subset \mathbb{R}^\ell$ is the correspondence of supply at the given price system p, which to every producer $b \in B$ assigns set $\eta_t^b(p)$ of production plans maximizing his profit, at the price system p, on the set $Q(Y^b, t)$,

$$\forall b \in B \;\; \eta_t^b(p) \stackrel{\text{def}}{=} \{Q(y^{b*}, t): p \circ y^{b*} = \max\{p \circ y^b : y^b \in Y^b\}\},$$

– $\pi_t: B \ni b \to \pi_t^b(p) \in \mathbb{R}$ is the maximal profit function at given price system p and

$$\pi_t^b(p) = p \circ Q(y^{b*}, t) \;\text{ where } y^{b*} \in \eta^b(p) \;\text{ for every } b \in B,$$

is called the modification of production system P, at time t, determined by vectors $q^1, \dots, q^k$.

Definition 3.5. The relational system

$$E_q(q^1, \dots, q^k; t) = (P_q(q^1, \dots, q^k; t), C_q, \theta, \omega)$$

is called the modification of economy E_q at time t, determined by vectors $q^1, \dots, q^k$. If E_q is the Debreu economy ($E_q = E_p = (\mathbb{R}^\ell, P, C, \theta, \omega)$), then the relational system

$$E_p(q^1, \dots, q^k; t) = (P(q^1, \dots, q^k; t), C, \theta, \omega)$$

is called the modification of economy E_q, at time t, determined by vectors $q^1, \dots, q^k$.

It is apparent that if economy E_q satisfies the condition (14), then for every $t \in [0,1]$, economy $E_q(q^1, \dots, q^k; t)$ is the economy reduced to subspace V.

Let V be a linear subspace of $\mathbb{R}^\ell$ given, in the sense of condition (11), by functionals $\tilde{g}^1, \dots, \tilde{g}^k$ of the form (12). Let $p \in \mathbb{R}^\ell$ and $E_q = (\mathbb{R}^\ell, P_q, C_q, \theta, \omega)$ be a private ownership economy (see def. 2.5).

In this situation the following is true:

Theorem 3.6. Assume that $p \notin V^T$ and vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ satisfy (28). Let $\tilde{Q}$ be an operator of the form (24) determined by vectors $q^1, \dots, q^k$.

1. If the sequence $((\hat{x}^a)_{a\in A}, (\hat{y}^b)_{b\in B}, p)$ (see (8)) is the state of quasi-equilibrium in economy E_q and

$$\sum_{a\in A} \hat{x}^a - \omega \in V, \tag{31}$$

then the sequence

$$\left((\hat{x}^a)_{a\in A}, \left(\tilde{Q}(\hat{y}^b, t)\right)_{b\in B}, p\right) = (\hat{x}^{a_1}, \dots, \hat{x}^{a_m}, \tilde{Q}(\hat{y}^{b_1}, t), \dots, \tilde{Q}(\hat{y}^{b_n}, t), p) \tag{32}$$

is the state of quasi-equilibrium in the private ownership economy $E_q(q^1, \dots, q^k; t)$.

2. If E_q is the Debreu economy, $E_q = E_p$, where the sequence $((x^{a*})_{a\in A}, (y^{b*})_{b\in B}, p)$ (see (9)) is the state of equilibrium in economy E_p and

$$\sum_{a\in A} x^{a*} - \omega \in V, \tag{33}$$

then the sequence

$$\left((x^{a*})_{a\in A}, \tilde{Q}(y^{b*}, t)_{b\in B}, p\right) = (x^{a_1*}, \dots, x^{a_m*}, \tilde{Q}(y^{b_1*}, t), \dots, \tilde{Q}(y^{b_n*}, t), p) \tag{34}$$

is the state of equilibrium in Debreu economy $E_p(q^1, \dots, q^k; t)$.

Proof.

1. Let $t \in [0,1]$ be given. By (27)

$$\forall b \in B\ \forall t \in [0,1]\ \ p \circ \hat{y}^b = p \circ \tilde{Q}(\hat{y}^b, t).$$

Hence, wealth $w(a)$ (see (5)) of every consumer $a \in A$ remains unchanged. Consequently condition

$$\forall\, a \in A\ \ p \circ \hat{x}^a \le p \circ e(a) + \sum_{b\in B} \theta(a,b) \cdot (p \circ \tilde{Q}(\hat{y}^b, t))$$

is valid. By (7) and (31) we get that:

$$\sum_{b\in B} \hat{y}^b = \sum_{a\in A} \hat{x}^a - \omega \in V.$$

The linearity of the mapping $\tilde{Q}(\cdot,t)$ implies that

$$\tilde{Q}(\sum_{b\in B}\hat{y}^b,t)=\sum_{b\in B}\tilde{Q}(\hat{y}^b,t)$$

and

$$\sum_{a\in A}\hat{x}^a-\sum_{b\in B}\tilde{Q}(\hat{y}^b,t)=\omega. \tag{35}$$

From the above we infer that the first condition by the thesis of the theorem is fulfilled.

2. Let $t\in[0,1]$ be given. By theorem 3.3, vector $\tilde{Q}(y^{b*},t)$ maximizes at price p the profit of every producer b on the production set $\tilde{Q}(Y^b,t)$. By (27), the wealth

$$w(a)=p\circ e(a)+\sum_{b\in B}\theta(a,b)\cdot(p\circ y^{b*})$$

(see (5)) of every consumer remains unchanged. Consequently, the consumers' budgets sets are the same as in the initial economy. In this situation the inequality

$$\forall\, a\in A\quad p\circ x^{a*}\le p\circ e(a)+\sum_{b\in B}\theta(a,b)\cdot(p\circ\tilde{Q}(y^{b*},t)),$$

is satisfied. Hence vector x^{a*} also maximizes, at price p, the preference of every consumer a on the budget set $\beta^a(p)$. By (10) and (33) the following may be easily inferred:

$$\sum_{b\in B}y^{b*}=\sum_{a\in A}x^{a*}-\omega\in V.$$

By the linearity of the mapping $\tilde{Q}(\cdot,t)$ we get that

$$\sum_{a\in A}x^{a*}-\sum_{b\in B}\tilde{Q}(y^{b*},t)=\omega \tag{36}$$

is valid, which gives the result.

□

Now, we have:

Theorem 3.7. Let $\tilde{Q}$ be an operator of the form (24) determined by vectors $q^1,\dots,q^k$ satisfying (23). Assume that condition (14) is satisfied in economy E_q, $p\in V^T$ and

$$\forall a\in A\ \ e(a)\in V. \tag{37}$$

1. If the sequence $((\hat{x}^a)_{a\in A}, (\hat{y}^b)_{b\in B}, p)$ (see (8)) is the state of quasi-equilibrium in economy E_q, then the sequence $\left((\hat{x}^a)_{a\in A}, \left(\tilde{Q}(\hat{y}^b, t)\right)_{b\in B}, p\right)$ (see (32)) is the state of quasi-equilibrium in private ownership economy $E_q(q^1, \dots, q^k; t)$.
2. If E_q is the Debreu economy, $E_q = E_p$, where the sequence $((x^{a*})_{a\in A}, (y^{b*})_{b\in B}, p)$ (see (9)) is the state of equilibrium in economy Debreu E_p, then the sequence $\left((x^{a*})_{a\in A}, \tilde{Q}(y^{b*}, t)_{b\in B}, p\right)$ (see (34)) is the state of equilibrium in Debreu economy $E_p(q^1, \dots, q^k; t)$.

Proof. Fix $t \in [0,1]$. By assumptions (14), (37) and formulas (2)–(5), keeping in mind that $p \in V^T$, we get that

$$\forall\, a \in A \;\; \beta^a(p) = X^a \;\text{ and }\; \forall\, a \in A \;\; e(a) \in X^a \subset V.$$

Moreover,

$$\forall t \in [0,1] \;\; \forall b \in B \;\; \forall y^b \in Y^j \;\; p \circ \tilde{Q}(y^b, t) = 0.$$

Hence, the condition

$$\forall\, a \in A \quad 0 = p \circ \tilde{x}^a \le p \circ e(a) + \textstyle\sum_{b\in B} \theta(a,b) \cdot (p \circ \tilde{Q}(\hat{y}^b, t)),$$

or

$$\forall\, a \in A \quad 0 = p \circ x^{a*} \le p \circ e(a) + \textstyle\sum_{b\in B} \theta(a,b) \cdot (p \circ \tilde{Q}(y^{b*}, t))$$

is satisfied, respectively. The conditions (35) or (36) can be proved in the same way as in the proof of theorem 3.6, which completes the proof.

□

The immediate consequence of theorems 3.6 and 3.7 is the following:

Theorem 3.8. Let $p \in \mathbb{R}^\ell$ and $E_p = (\mathbb{R}^\ell, P, C, \theta, \omega)$ be the Debreu economy satisfying conditions (14) and (37) with a proper subspace $V \subset \mathbb{R}^\ell$ of the form (11). There is an operator $\tilde{Q}$ of the form (24) such that if the sequence $((x^{a*})_{a\in A}, (y^{b*})_{b\in B}, p)$ (see (9)) is the state of equilibrium in economy E_q, then the sequence $\left((x^{a*})_{a\in A}, \tilde{Q}(y^{b*}, t)_{b\in B}, p\right)$ (see (34)) is the state of equilibrium in Debreu economy $E_p(q^1, \dots, q^k; t)$.

Proof. The proof goes in the same way as the proofs of the second parts of the thesis of theorems 3.6 and 3.7.

□

Notice that, the quasi-production system $P_q(q^1, \dots, q^k; 1)$ – the component of the economy $E_q(q^1, \dots, q^k; 1)$, is the image of the quasi-production system P_q – the component of economy E_q by projection $Q(\cdot) = \tilde{Q}(\cdot, 1)$ (see (25)) determined by vectors $q^1, \dots, q^k$. In this sense we say that both: the quasi-production system $P_q(q^1, \dots, q^k; 1)$ and the economy $E_q(q^1, \dots, q^k; 1)$ are determined by vectors $q^1, \dots, q^k$ (def. 3.5).

Remark 3.9. Consider a Debreu economy $E_p = (\mathbb{R}^\ell, P, C, \theta, \omega)$ satisfying conditions (14) and (37) with a subspace $V \subset \mathbb{R}^\ell$. Let vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ determine mapping $\tilde{Q}$ by the thesis of theorem 3.8. Theorem 3.8 guarantees the existence of equilibrium in every economy $E_p(q^1, \dots, q^k; t)$ for $t \in [0,1]$ if equilibrium exists in initial economy E_p. Observe that mapping $\breve{Q}: \mathbb{R}^\ell \times \mathbb{R}_+ \to \mathbb{R}^\ell$

$$\breve{Q}(x, t) = x - t \cdot \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s$$

is the semi-dynamical system (see Sibirskij, Szube, 1987) in space $\mathbb{R}^\ell$. For every $t \geq 0$, the production system $P(q^1, \dots, q^k; t)$ (see def. 3.4) is, besides price system p, the image of production system P from economy, by mapping $\breve{Q}(\cdot, t)$. Similarly, the relational system

$$E_p(q^1, \dots, q^k; t) = (P(q^1, \dots, q^k; t), C, \theta, \omega),$$

is the Debreu economy. If p is the equilibrium price vector in economy E_p then sequence

$$(x^{a_1*}, \dots, x^{a_m*}, \breve{Q}(y^{b_1*}, t), \dots, \breve{Q}(y^{b_n*}, t), p^*) \in (\mathbb{R}^\ell)^{m+n+1}$$

is the state of equilibrium in economy $E_p(q^1, \dots, q^k; t)$. Hence, mapping $\breve{Q}$ lets us put the whole systems of economies $\{E_p(q^1, \dots, q^k; t) : t \geq 0\}$ "in motion", where variable t means time. In the course of that motion, the production system P from economy E_p is changed in time, but the rest of relational systems in every economy $E_p(q^1, \dots, q^k; t)$: $t \geq 0$ are not changed. At $t = 1$ economy $E_p(q^1, \dots, q^k; 1)$ is, besides the equilibrium price system (which may be but not necessary), contained in the subspace V.

The recipe for producers' adjustment trajectory can be forced by the market or it can be set and driven by a person or an institution. If each producer modifies his activities according to the same trajectory of the form (24), then the transformation of the production sector with keeping equilibrium in the economy will be successful. If some of producers choose a different trajectory than the others do, then generally (despite particular cases) equilibrium will not exist at point $t = 1$. In summary, the potential producers' disagreement on the choice of the trajectory (24) or the exclusion of even one producer from the modification process may cause disequilibrium in the economy at point $t = 1$.

4. THE OPTIMAL PRODUCERS' ADJUSTMENT TRAJECTORY

Now, let us focus on the comparison of producers' adjustment trajectories as well as on the characterization of the trajectory under study, optimal under the criterion of distance minimizations.

Let E_q be a private ownership economy (see def. 2.5) and $p \in \mathbb{R}^\ell$ be the price system in economy E_q. Consider a proper linear subspace $V \subset \mathbb{R}^\ell$, $V \neq \{0\}$ defined as in (11). Let us notice that the system of equalities (23) has only one solution if and only, if $k = \ell$ (ℓ – number of commodities, k – number of functionals describing subspace V). Hence, for $V \neq \{0\}$ the number of functionals defining subspace V (see (11)) is less than the number of commodities ($k < l$). If $k \in \{1, \dots, \ell - 1\}$ then system of equalities (23) has infinitely many solutions. If $p \notin V^T$, then system of equalities (28) has only one solution if and only, if $k = \ell - 1$. If $k \in \{1, \dots, \ell - 2\}$ then the system of equalities (23) has infinitely many solutions. Hence, the producers, who want to change their production plans, have often infinitely many possibilities of choice of trajectories of the form (24).

Assuming that producers want to, or have to change their production activity, other problems (questions) arise. How to compare producers' adjustment trajectories? Which of them are the best or satisfactory enough for the producers? This certainly depends on the criterion of the choice. We assume that the producers, keeping in mind the necessity of modifying their plans to plans contained in subspace V, want also to minimize the costs. It results in changing the activities of producers as little as possible. It means that the difference between the respective coordinates of every production plan and its modification will be also properly small. Moreover, the producers' plans contained in subspace V should remain unchanged. Hence, we determine for every $x = (x_1, \dots, x_\ell) \in \mathbb{R}^\ell$, the norm

$$\|x\| = max\{|x_l|: l \in \{1,2, \dots, \ell\}\}. \quad (38)$$

Assume that the given subspace $V \subset \mathbb{R}^\ell$ is defined (see (11)) by functionals $\tilde{g}^1, \dots, \tilde{g}^k$ of the form (12). Let $Q \in \mathcal{P}(\mathbb{R}^\ell, V)$, $Q(x) = x - \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s$ be the projection (see (25)) determined by vectors $q^1, \dots, q^k$ satisfying (23). It is well known (see Cheney, 1966) that, for every $x \in \mathbb{R}^\ell$

$$dist(x, V) \leq \|(Id - Q)(x)\| \leq \|Id - Q\| dist(x, V), \quad (39)$$

where

$$\|Id - Q\| = \sup\{\|(Id - Q)(x)\|: x \in \mathbb{R}^\ell \wedge \|x\| \leq 1\}. \quad (40)$$

The number $\|Id - Q\|$ can be interpreted as the distance (by (39)) between the initial economy E_q and its final modification $E_q(q^1, \dots, q^k; 1)$. Hence the projection Q is identified with the producers' adjustment trajectory $\tilde{Q}(x,t) = x - t \cdot \sum_{s=1}^{k} \tilde{g}^s(x) \cdot q^s$ (see (24)). By this reason, the projection Q is also called the producers' adjustment trajectory and the number $\|Id - Q\|$ is called the coefficient of the change of the economy E_q determined by trajectory Q. We also say that the projection Q realizes the number $\|Id - Q\|$.

It is apparent, by (39) and (40), that $\|Id - Q\| \geq 1$. Moreover, if the norm $\|Id - Q\|$ is not large, then the production plans and their modifications are close in terms of distance. Hence, the mapping $Q \in \mathcal{P}(\mathbb{R}^\ell, V)$ for which number $\|Id - Q\|$ is the smallest possible, is the producers' adjustment trajectory optimal (best) under the criterion of distance minimization. It will be called the optimal producers' adjustment trajectory.

Keeping in mind the motivations of producers and properties of projections presented above, we define the preference relation of producer $b \in B$ in the set $\mathcal{P}(\mathbb{R}^\ell, V)$. Let $b \in B$ and $V \subset \mathbb{R}^\ell$, $V \neq \{0\}$ be a subspace of the form (11), defined by functionals $\tilde{g}^1, \ldots, \tilde{g}^k$ given by (12). Let us notice, that without loss of generality we can assume that for every $s \in \{1, \ldots, k\}$, where $k \in \{1, \ldots, \ell - 1\}$, $\sum_{l=1}^{\ell} |g_l^s| = 1$. Define, for every functional $\tilde{g}^s$, $s \in \{1, \ldots, k\}$, the set

$$supp\tilde{g}^s = \{l \in \{1, \ldots, \ell\}: g_l^s \neq 0\}$$

The changes in producers' activities, which imply that condition (15) is satisfied, are called the adjustment of technologies to subspace V. It is said that a producer b is neutral to the adjustment of technologies to subspace V, if

$$l \in \bigcup_{s=1}^{k} supp\tilde{g}^s \Longrightarrow \forall y^b \in Y^b \; y_l^b = 0. \tag{41}$$

So, the initial plans of producers, who are neutral to the adjustment of technologies to subspace V, are contained in V. The set of producers neutral to the adjustment of technologies to subspace V will be denoted by B_0.

To model the changes in the production system relaying on the adjustment of technologies to subspace V, it is worth assuming that there is at least one producer who is not neutral to the adjustment of technologies to V ($B \backslash B_0 \neq \emptyset$) as well as that every producer from set $B \backslash B_0$ will modify his production plans under the criterion of distance minimization. Let $Q_1, Q_2 \in \mathcal{P}(\mathbb{R}^\ell, V)$. Taking the above criterion into consideration, the preference relation of producer $b \in B \backslash B_0$ in set $\mathcal{P}(\mathbb{R}^\ell, V)$ is defined as follows

$$Q_2 \preccurlyeq^b Q_1 \Leftrightarrow \|Id - Q_1\| \leq \|Id - Q_2\|. \tag{42}$$

Hence, for every producer from set $B \backslash B_0$ is assigned the same preference relation of the form (42). By the fact that every producer $b \in B_0$ will not change his production set, all the producers' adjustment trajectories (projections from set $\mathcal{P}(\mathbb{R}^\ell, V)$) are indifferent for him. The above indifference will be traditionally marked, for $Q_1, Q_2 \in \mathcal{P}(\mathbb{R}^\ell, V)$, by

$$Q_2 \sim^b Q_1. \tag{43}$$

Condition (43) means that

$$Q_2 \preccurlyeq^b Q_1 \quad \text{and} \quad Q_1 \preccurlyeq^b Q_2$$

for $b \in B_0$. Let us recall that every producer $b \in B \backslash B_0$ wants to, or has to modify his production plans under the criterion of distance minimization. Additionally, according to the rationality assumption he will choose the optimal producers' adjustment trajectory provided such exists. Combining conditions (42) and (43), we define the preference relation $\preccurlyeq$ in set $\mathcal{P}(\mathbb{R}^\ell, V)$ by the rule

$$Q_2 \preccurlyeq Q_1 \Leftrightarrow \forall b \in B \;\; Q_2 \preccurlyeq^b Q_1. \tag{44}$$

The relation $\preccurlyeq$ defined in (44) is the producers' preference relation in the set of defined producers' adjustment trajectories.

We will explain that the producers' preference relation defined in (44) will have a maximal element. Notice that, the dimension of commodity-price space $\mathbb{R}^\ell$ is finite, then the problem of the distance minimization in set $\mathcal{P}(\mathbb{R}^\ell, V)$ has a solution (see for example Cheney, 1966). Precisely, there is a projection $Q_0 \in \mathcal{P}(\mathbb{R}^\ell, V)$ such that

$$\|Id - Q_0\| = inf\{\|Id - Q\|: Q \in \mathcal{P}(\mathbb{R}^\ell, V)\} \tag{45}$$

(see Lewicki Odyniec, 1990). The projection $Q_0 \in \mathcal{P}(\mathbb{R}^\ell, V)$ satisfying (45), is called the cominimal projection (see Lipieta, 1999). It is obvious that the cominimal projection Q_0 minimizes the distance between the initial economy E_q and its final modifications $E_q(q^1, \dots, q^k; 1)$. That means that Q_0 is the maximal element of the producers' preference relation defined in (44) and it minimizes the coefficient of the change of the economy.

Unfortunately, the formula for cominimal projections as well as the number (45) are not known besides some cases. However, if properties (18) or (22) are fulfilled, then the problem of indicating such cominimal projection Q_0, for which $\|Id - Q\| = 1$, becomes quite simple. Namely, the following is true:

Theorem 4.1 (theorem 3.1 in Lipieta, 1999). Let $V \subset \mathbb{R}^\ell$, $V \neq \{0\}$ be a linear subspace of the form (11) defined by functionals $\tilde{g}^1, \dots, \tilde{g}^k$ of the form (12) satisfying, for every $s \in \{1, \dots, k\}$, condition $\sum_{l=1}^{\ell} |g_l^s| = 1$. Then

- $\|Id - Q\| = 1$ if and only if $\bigcap_{s=1}^{k} supp\tilde{g}^s = \emptyset$.
- if $\|Id - Q\| = 1$, then the cominimal projection Q_0 is determined by vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ such that if $g_l^i \neq 0$ for some $i \in \{1, \dots, k\}$ and $l \in \{1, 2, \dots, \ell\}$, then for $s \in \{1, \dots, k\}$

$$q_l^s = \begin{cases} 1 & \text{for } s = i \wedge g_l^i > 0, \\ -1 & \text{for } s = i \wedge g_l^i < 0, \\ 0 & \text{for } s \neq i. \end{cases} \quad (46)$$

The projection Q_0 determined by vectors $q^1, \dots, q^k \in \mathbb{R}^\ell$ of the form (4.9), under the assumptions that $\bigcap_{s=1}^{k} supp\tilde{g}^s = \emptyset$ as well as, for every $s \in \{1, \dots, k\}$, $\sum_{l=1}^{\ell} |g_l^s| = 1$, is the closest to identity mapping. Then, changes in the production sector induced by projection Q_0 are the least, among changes determined by other projections.

Consider a subspace $V \subset \mathbb{R}^\ell$ defined as in (3.11) with functionals $\tilde{g}^1, \dots, \tilde{g}^k$ of the form (12). Let $\bigcap_{s=1}^{k} supp\tilde{g}^s = \emptyset$ and for every $s \in \{1, \dots, k\}$, $\sum_{l=1}^{\ell} |g_l^s| = 1$. Assume that the initial Debreu economy $E_p = (\mathbb{R}^\ell, P, C, \theta, \omega)$ satisfies assumption α_1 or α_2, where

– α_1: $p \notin V^T$, $q^1, \dots, q^k$ are the solution of (28) as well as (33) is fulfilled,
– α_2: $p \in V^T$, $q^1, \dots, q^k$ satisfy (3.13) as well as conditions (14) and (37) are fulfilled.

Then projection Q determined by vectors $q^1, \dots, q^k$ "moves" the economy E_p from its initial equilibrium state into another equilibrium state. On the basis of the above, if vectors $q^1, \dots, q^k$ satisfy additionally condition (46), then projection Q determined by vectors $q^1, \dots, q^k$, projection is the optimal producers' adjustment trajectory, $Q = Q_o$ which gives the least coefficient of the change of the considered economy. Similarly, mapping $\tilde{Q}_o$ of the form (24), determined by vectors of the form (46), is also the optimal producers' adjustment trajectory. Moreover, for every $t \in [0,1]$, the production system $P(q^1, \dots, q^k; t)$ (see def. 3.4) as well as the whole economy $E_p(q^1, \dots, q^k; t)$ is the image, by the mapping $\tilde{Q}_o$, of the initial production system P or the private ownership economy E_p respectively.

If vectors $q^1, \dots, q^k$ of the form (46) are not orthogonal to the price vector $p \notin V^T$ (it means that condition (28) and consequently assumption α_1 are not satisfied), then there is no equilibrium in economy $E_q(q^1, \dots, q^k; 1)$, despite some particular cases. Moreover, the economy $E_q(q^1, \dots, q^k; 1)$ (see def. 2.5 and 3.5) does not have to be the Debreu economy.

Let us emphasize that if a producer, who is not neutral to the adjustment of technologies to subspace V, does not change his production plans according to mapping $\tilde{Q}$ of the form (24), then equilibrium could not exist at least at one $t \in (0,1]$. If every producer from set $B \backslash B_0$, under the assumption α_1 or α_2, follows the same trajectory $\tilde{Q}$ as well as the producers from set B_0 do not change their production plans, then there will be equilibrium in the economy with modified production system if it existed in the initial economy (see also theorem 3.8).

Let us notice that either new firms or commodities do not appear and are not eliminated from the producers' activities in the considered modifications of the economy under study. Moreover, the technologies are mildly modified as well as the prices are not changed. These result in the same profits. Hence, the production sector of the discussed economy evolves in the framework of the Schumpeterian circular flow (see Schumpeter, 1912; Lipieta, 2013).

5. CONCLUSIONS

The results of this research lead to simplifying the geometric structure of the initial economy. It is caused by the appearance of the linear dependency between quantities of some commodities in all producers' plans in the modified form of the economy or by the elimination of some harmful commodities from the production processes. Consequently, the correspondences and functions – the components of the final production system – depend, in fact, on fewer variables than in the beginning.

On the basis of the above, the prerequisites for the appearance of the optimal producers' adjustment trajectory under the criterion of distance minimization, with the smallest possibly the coefficient of the change of the considered economy, were presented. The definition of the mentioned trajectory also has been formulated.

If the changes of producers' activities are caused by other reasons than those considered in the paper, or the criterion for comparing producers' adjustment trajectories is different, then the recipe for producers' adjustment trajectories as well as the optimal producers' adjustment trajectories might be modified. The studies on designing the producers' adjustment trajectories, under other criterions, still remains within our research plans.

REFERENCES

Aliprantis C. D., (1996), *Problems in Equilibrium Theory*, Springer – Verlag Berlin, Heidelberg, German.

Cheney E. W., (1966), *Introduction to Approximation Theory*, Mc Grow Hill, New York.

Debreu G., (1959), *Theory of Value*, New York, Viley.

Lewicki G., Odyniec W., (1990), *Minimal Projections in Banach Spaces*, Lecture Notes in Mathematics, 1449, Springer-Verlag, Berlin/New York.

Lipieta A., (1999), Cominimal Projections in . *Journal of Approximation Theory*, 98, 86–100.

Lipieta A., (2010), The Debreu Private Ownership Economy with Complementary Commodities and Prices, *Economic Modelling*, 27, 22–27.

Lipieta A., (2012), The Economy with Production and Consumption Systems Changing in Time, *Przegląd Statystyczny*, 59 (3), 233–245.

Lipieta A., (2013), Mechanisms of Schumpeterian Evolution**,** in: Malawski A., (ed.), *Innovative Economy as the Object Investigation in Theoretical Economics*, Cracow University of Economics Press, 94–119.

Magill M., Quinzii M., (2002), *Theory of Incomplete Markets*, MIT Press, Cambridge.

Mas-Colell A., Whinston M. D., Green J. R., (1959), *Microeconomic Theory*, Oxford University Press, New York.

Moore J., (2007), *General Equilibrium and Welfare Economics*, Springer Berlin-Heidelberg-New Jork.

Radner R., (1972), Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets, *Econometrica*, 40 (2), 289–303.

Sibirskij K. S., Szube A. S., (1987), *Semidynamical Systems* (Topological Theory). Russian, Sztinica, Kiszyniów, Belknap Press of Harvard University Press.

Schumpeter J. A., (1912), *Die Theorie der Wirtschaftlichen Entwicklung*, Leipzig, Duncker & Humblot; English translations, The Theory of Economic Development, Cambridge, MA, Harvard University Press 1934 and A Galaxy Book, New York, Oxford University Press 1961.

Varian H. R., (1999), *Intermediate Microeconomics, A Modern Approach.* W. W. Norton & Company, New York, London.

OPTYMALNA TRAJEKTORIA DOSTOSOWAWCZA PRODUCENTÓW

Streszczenie

W artykule została zdefiniowana grupa ścieżek dostosowawczych (trajektorii), które opisują niezbędne zmiany w sferze produkcji, spowodowane koniecznością lub chęcią producentów dostosowania swojej działalności na rynkach do danych wymogów. Działalność producentów jest modelowana w ekonomii Debreu, a wymogi są zadane analitycznie przez funkcjonały liniowe.

Rozważane trajektorie ilustrują zmiany w systemie produkcji, które nie zaburzają równowagi w ekonomii w okresie transformacji, chociaż początkowy stan równowagi może ulec zmianie. W zbiorze omawianych trajektorii została zdefiniowana relacja preferencji, której element maksymalny tzw. optymalna trajektoria dostosowawcza producentów, przy pewnych założeniach, wyznacza kierunek najbardziej korzystnych dla producentów zmian, z punktu widzenia minimalizacji strat.

Słowa kluczowe: ekonomia z własnością prywatną, równowaga, zbiory liniowe, projekcje

THE OPTIMAL PRODUCERS' ADJUSTMENT TRAJECTORY

Abstract

The trajectories illustrating the necessary changes in the production sphere, which are caused by the necessity or the wish of producers, who adjust their activities to the given requirements, are analyzed in the paper. The producers' activities are modeled in the Debreu economy, while the requirements are given analytically, by using the linear functionals.

If the producers change their plans of action due to the considered trajectories, equilibrium in the economy will be kept, although the initial state of equilibrium can be replaced by the other one.

In the set of trajectories under study, the preference relation is defined. Under some assumptions, the maximal element of the above relation, so called the best producers' adjustment trajectory, indicates the best path of changes in producers' activities, under the criterion of losses minimization.

Keywords: private ownership economy, equilibrium, linear sets, projections