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## LATENT VARIABLE MODELS – ISSUES ON MEASUREMENT AND FINDING EXACT CONSTRUCTS IN CUSTOMERS' VALUES

### 1. INTRODUCTION TO MEASUREMENT MODEL AND LATENT CONSTRUCT

In some accounts, measurement can be defined as the assignment of numbers to categories of observations. The properties of the numbers become the properties of the measurement – nominal, ordinal, interval, ratio, and so on [17]. In measurement model the inference is to relate the scores to the construct (here otherwise called latent variable or latent construct). The measurement model must help in understanding and evaluation the scores that come from the item responses. In short they should tell us about the construct, and the use of the results in practical applications. Simply put, the measurement model must translate scored responses to locations on the construct (variable) map. Some examples of measurement models are:

- the “true – score” model of classical test theory,
- the “domain score” model,
- factor analysis models,
- item response models,
- latent class models.

Typically we have two types of models. The first approach focuses on the items and their relationship to the construct [5; 6]. The second approach focuses on the scores and their relationship to the construct. The intuitive foundation of the instrument-focused approach is what might be called simple score theory. There needs to be some sort of an aggregation of information across the items, but the means of aggregation is either left vague or assumed on the basis of historical precedent to be the summation of item scores. Simple score theory is more like a folk theory, but nevertheless exerts a powerful influence on intuitive interpretations.

The simple score theory approach was formalized by classical test theory (also known as true score theory). This approach was founded by Spearman [14; 15; 16]. He borrowed a perspective from the fledgling statistical approach of the time and posited that an observed total score on the instrument  $X$ , was composed of the sum of a “true score”  $T$  and an “error”  $E$ . The introduction of an error term allows for a quantification of inconsistency in observed scores, which is part of the solution to the problem with Guttman scales. Guttman scaling focused attention on the meaningfulness of the results from the instrument (i.e., its validity), whereas classical test theory models the statistical nature of the scores focused attention on the consistency

of the results from the instrument (i.e., its reliability). There has been a long history of attempts to reconcile these two approaches. One notable early approach is that of Thurstone [22], who clearly saw the need to have a measurement model that combined the virtues of both.

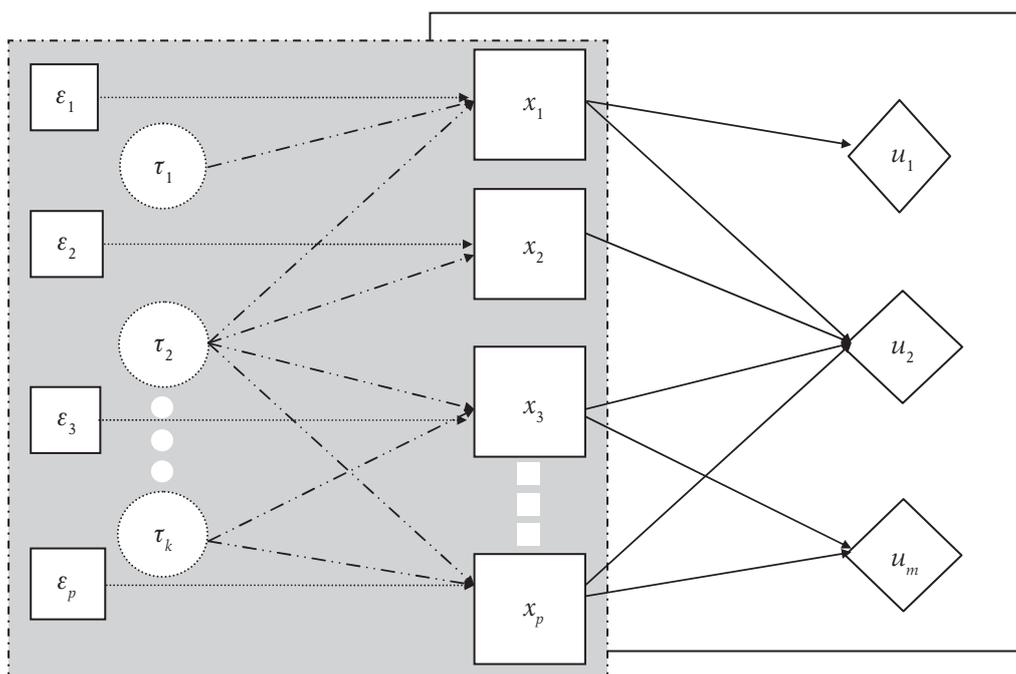


Figure 1. Measurement model and scale

Source: [21].

In statistically defined model, it must be assumed that  $x$  will be a vector of order  $p$  of the observed variables. The measurement model is therefore:

$$x = B\tau + \varepsilon, \tag{1}$$

where  $\tau$  is a vector of order  $k$  of the true scores,  $B$  is a  $p \times k$  pattern matrix which defines the relationship between  $x$  and  $\tau$ , and  $\varepsilon$  is a vector of the measurement errors. The true scores and the measurement errors are not directly observable and must be estimated from the data. The assumptions are analogous to the classical test theory which was:

$$E(\tau) = \mu, E(\varepsilon) = 0, \text{cov}(\tau, \varepsilon) = 0, (\tau) = \Phi \text{ and } \text{cov}(\varepsilon) = \Psi. \tag{2}$$

Thus, the covariance structure of the observed variables is [15]:

$$\text{cov}(x) = E(B\tau + \varepsilon)(B\tau + \varepsilon)' = B\Phi B' + \Psi = \Sigma. \quad (3)$$

The covariance matrix  $\Sigma$  is assumed to be non-singular and positively definite:

$$\text{rank}(\Sigma) = p \text{ and } \Sigma > 0. \quad (4)$$

Unlike, the measurement scale (otherwise called latent construct) is a linear combination of the items. The collections of items into a composite score is intended to reveal levels of theoretical variables not readily observable by direct means (often referred to as scale). We develop scales when we want to measure phenomena that we believe to exist because of our theoretical understanding of this world, but that we can not assess directly. In general, we have  $m$  scales as a vector  $u = A'x$ , where  $A$  is a  $p \times m$  matrix of the weights. The case of one scale is denoted by  $u = a'x$  where  $a$  is a vector of the weights. It is important to distinguish between the concepts of the measurement scale from the measurement model. The model discriminates the underlying structure of the measurement from the use of the items [figure 1]. The scales weighted by the corresponding pattern elements of the model, denoted by  $u = B'x$  are termed true score images. They are used to assess the structural validity of the measurement model.

## 2. ADJUSTMENTS IN THE MEASUREMENT MODEL DEPENDING ON DISTANCE BETWEEN RESPONDENT AND RESPONSE

There are basic assumptions as for the corrected model. First of all the measurement model must enable one firstly to interpret the distance between respondent and response on the construct (variable) map and secondly the measurement model must enable one to interpret distance between different responses on the construct map, and also the difference between different respondents. In order to find sense between these above two, we should have looked into what distance might mean in the context of a construct map.

On a construct map, distance between respondents and responses indicates the probability of making that response. To express this as an equation, we assume the probability of the response ( $\text{Pr}(\text{response})$ ) is given by some function  $f$  of the difference between the respondent and the response [24]:

$$\text{Pr}(\text{response}) = f(q - d). \quad (5)$$

where interpretation for this formula can be given as follows:

1. zero distance between person and response would mean that person is likely to endorse statement with a certain probability,
2. respondent *above* response would indicate a greater probability,
3. respondent *below* response would indicate a lesser probability.

Hence, we can say that the model must have qualitative features from 1 to 3. However, these qualitative features are not sufficient to preserve the idea of a “map.” For that the requirements 2 and 3 would need to be more detailed, giving a specific metric form.

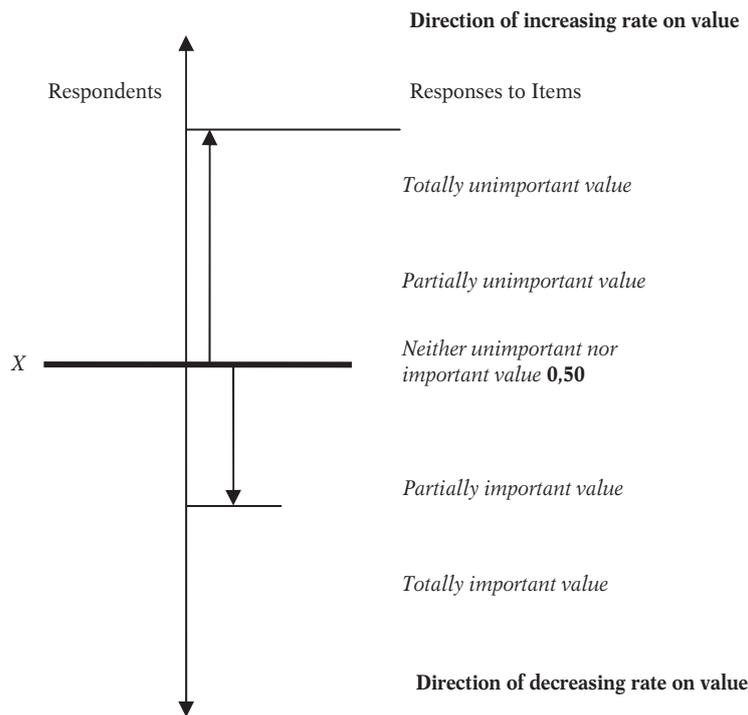


Figure 2. Different item locations with respect to a single person

Source: [24].

To consider distance between person and item, we can interpret it by the following figure 2. This figure illustrates the situation for one person approximately in the middle of the construct. For this person, items that are at a similar level would be expected to elicit agreement at about 0,50 probability, whereas items that are above would tend to result in a positive response with a greater probability and the opposite for those below.

We can also consider distances between item responses themselves. Figure 3 illustrates the distance between two item responses, considered by two persons. One specific qualitative consequence of this is that the order (on the map) of the item responses must remain the same for all respondents, and the order of the respondents (on the map) must remain the same for all item responses.

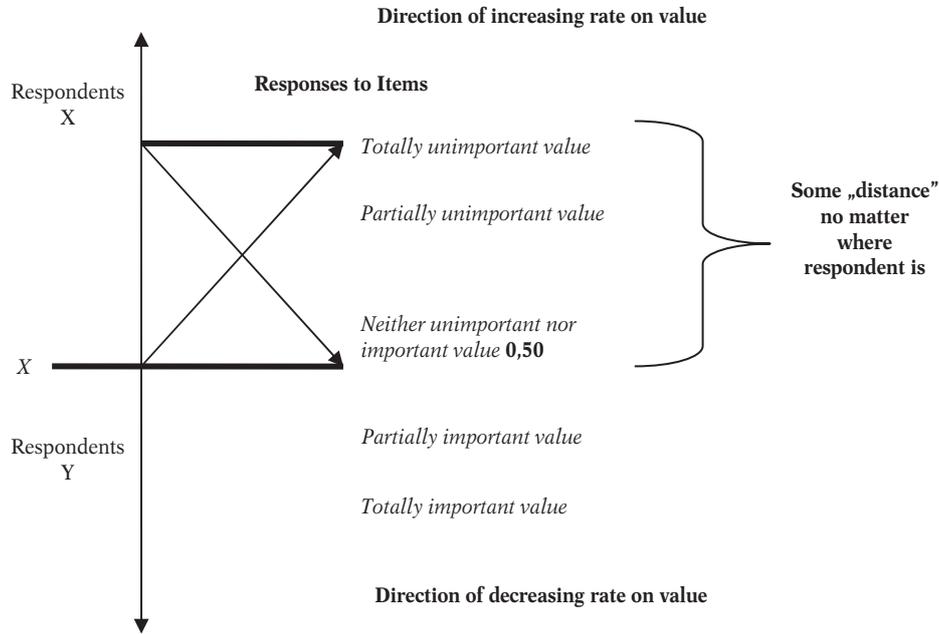


Figure 3. Different item locations with respect to two persons

### 3. FACTOR ANALYSIS IN THE DETECTION OF LATENT CONSTRUCT(S)

Factor analysis belongs to a group of latent variable models (table 1). The aim of factor analysis is to explain the outcome of  $p$  variables (items) in the data matrix using the so-called factors. Ideally all the information in data matrix can be reproduced by a smaller number of factors. These factors are interpreted as latent (unobserved) common characteristics of the observed variables. The case just described occurs when every observed  $x = (x_1, \dots, x_p)$  can be written as:

$$x_j = \sum_{\ell=1}^k q_{j\ell} f_{\ell} + \mu_j, \quad j = 1, \dots, p \quad (6)$$

Here  $f_{\ell}$  for  $\ell = 1, \dots, k$  denotes the factors. The number of factors,  $k$  should always be much smaller than  $p$ . It is then possible to create a representation of the observations that is similar to the one in equation (6) by means of principal components, but only if the last  $p - k$  eigenvalues corresponding to the covariance matrix are equal to zero. Considering a  $p$  - dimensional random vector  $X$  with mean  $\mu$  and covariance matrix  $\text{Var}(X) = \Sigma$  enables to form a model similar to (6) being written for  $X$  in matrix notation, namely

$$X = QF + \mu, \quad (7)$$

where  $F$  is the  $k$  – dimensional vector of the  $k$  – factors. When using the factor model (7) it is often assumed that the factors  $F$  are centered, uncorrelated and standardized:  $E(F) = 0$ . The existing factor model is defined by covariance matrix of model:

$$\Sigma = E(X - \mu)(X - \mu)^T = QE(FF^T)Q^T = QQ^T = \sum_{j=1}^k \lambda_j \gamma_j \gamma_j^T. \tag{8}$$

Table 1

Classification of latent variable models

Latent variables $f$	Observed variables $x$	
	Metrical (interval/ratio)	Categorical (nominal/ordinal)
Metrical (interval/ratio)	Factor analysis	Latent trait analysis
Categorical (nominal/ordinal)	Latent profile analysis	Latent class analysis

Source: [2; 3; 9; 10; 13].

It is also common in factor analysis to split the influences of the factors into: common and specific ones. There are, for example, highly informative factors that are common to all of the components of  $X$  and factors that are specific to certain components. The factor analysis model used in praxis is a generalization of (7):

$$X = QF + U + \mu \tag{9}$$

where:

$Q$  –  $(p \times k)$  matrix of the (non – random) loadings of the common factors  $F(k \times 1)$ ,  
 $U$  – matrix of the random specific factors.

It is also assumed that the factor variables  $F$  are uncorrelated random vectors and that the specific factors are uncorrelated and have zero covariance with the common factors. More precisely, it is assumed that:

$$\begin{aligned} E(F) &= 0, \\ \text{Var}(F) &= \tau_k, \\ E(U) &= 0, \\ \text{Cov}(U_i, U_j) &= 0, \\ \text{Cov}(F, U) &= 0. \end{aligned} \tag{10}$$

The generalized factor model (7) together with the assumptions given in (8) can be constituted:

$$\underset{(p \times 1)}{X} = \underset{(p \times k)}{Q} \underset{(k \times 1)}{F} + \underset{(p \times 1)}{U} + \underset{(p \times 1)}{\mu} \tag{11}$$

where:

$\mu_j$  – mean of variable  $j$ ,

$U_j$  –  $j$ -th specific factor,

$F_l$  –  $l$ -th common factor,

$q_{jl}$  – loading of the  $j$ -th variable on the  $l$ -th factor.

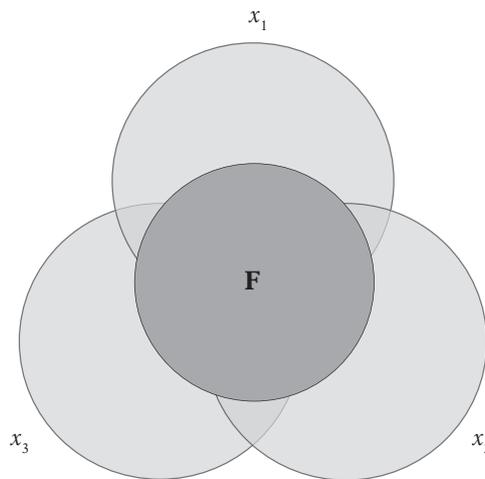


Figure 4. Depiction of the commonality among three measures of a single construct

The central concern of factor analysis in the process of modeling factors, is sometimes referred to as latent variables. Factors are influences that are not directly measured but account for commonality among a set of measurements. In figure 4 the commonality among measures of a construct is depicted in two ways. On the left, a diagram is used to illustrate a pattern of shared variance among scores on the three measures. Each circle represents the variance in one of the measures,  $x_1$ ,  $x_2$  and  $x_3$ . The overlap of the circles represents shared variance, or covariance. The shaded area, labeled “F”, represents the area of overlap involving all three circles. It is this area that corresponds to a factor or otherwise called “latent construct”.

#### 4. EFA AND CFA AS LATENT METRICAL VARIABLE MODELS

There are two latent models based on metrical characteristics: CFA (Confirmatory Factor Analysis) and EFA (Exploratory Factor Analysis). Confirmatory factor analysis (CFA), otherwise referred to as restricted factor analysis, structural factor analysis, or the measurement model, typically is used in a deductive mode to test hypotheses regarding unmeasured sources of variability responsible for the commonality among a set of scores. It can be contrasted with exploratory factor analysis (EFA) which addresses the same basic question but in an inductive, or discovery-oriented, mode.

In order to illustrate associations among the measured variables making up complementary latent constructs a path diagrams for CFA and EFA should be drawn. In the path diagram, the rectangles represent the measured variables, generically referred to as indicators. Ellipses represent unmeasured variables. The large ellipse represents a factor, whereas the smaller ellipses represent errors of measurement or uniqueness, which are unobserved sources of influence unique to each indicator. The single-headed arrows suggest causal influence, indicating specifically that each indicator is caused by two unmeasured influences:

- a causal influence it shares with the other indicators,
- an additional causal influence not shared with the remaining indicators.

The two-head curved arrows associated with the latent variables represent variances.

The path diagram can be translated directly into statistical form through a set of measurement equations, which specify the relations between factors and indicators. Consider the single equation, expressed in Bentler-Weeks [3] notation:

$$x_i = *F_1 + \dots + *F_j + u_i. \quad (12)$$

According to this equation, variability in the  $i$ th indicator is an additive function of  $j$  differentially weighted factors and the  $i$ th unique factor.

$$\begin{aligned} x_1 &= *F_1 + *F_2 + u_1 \\ x_2 &= *F_1 + *F_2 + u_2 \\ x_3 &= *F_1 + *F_2 + u_3 \\ x_4 &= *F_1 + *F_2 + u_4 \\ x_5 &= *F_1 + *F_2 + u_5 \\ x_6 &= *F_1 + *F_2 + u_6 \end{aligned} \quad (13)$$

Careful consideration of this equation illustrates a basic difference between CFA and EFA. In EFA, no restrictions are placed on the pattern of weights, denoted by “\*”, associated with the factors. As such, each indicator is, to some degree, influenced by each of the  $j$  factors and, in CFA language, is “free” to be estimated in the analysis. Thus, the EFA measurement equations for a two-factor model involving six indicators, displayed in figure 5.

In contrast, CFA requires the imposition of certain restrictions on the pattern of weights, or factor loadings, a seemingly limiting characteristic that, in fact, pays substantial inferential dividends. Below equations are the CFA measurement equations for a prototypic two-factor CFA model involving six indicators, shown also in path diagram form in figure 5.

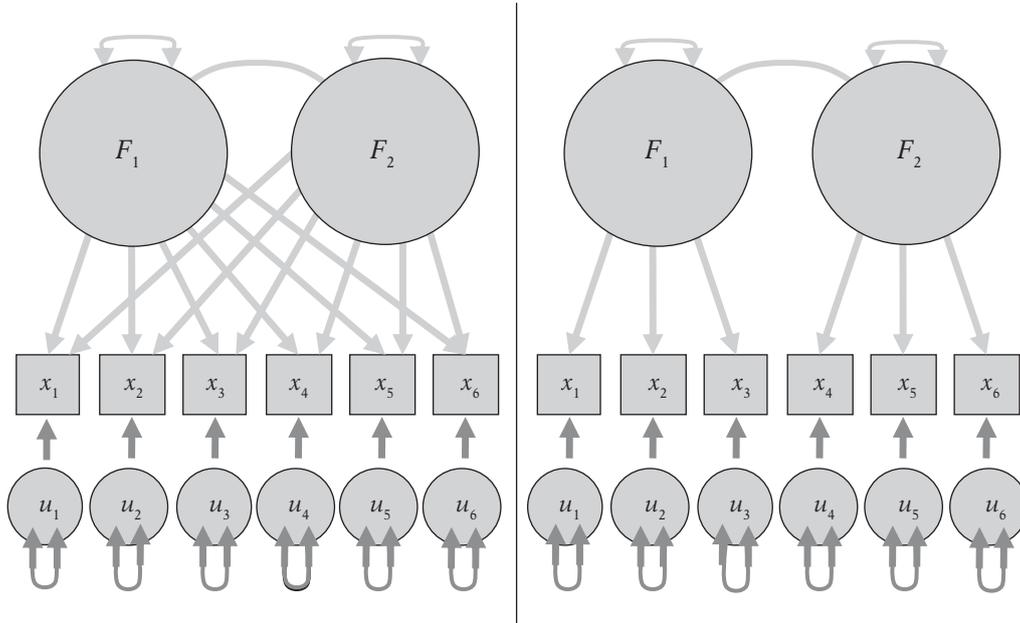


Figure 5. Path diagrams of two correlated factors as modeled using exploratory factor analysis (EFA) (left) and confirmatory factor analysis (CFA) (right).

$$\begin{aligned}
 x_1 &= *0F_1 + *0F_2 + u_1 \\
 x_2 &= *0F_1 + *0F_2 + u_2 \\
 x_3 &= *0F_1 + *0F_2 + u_3 \\
 x_4 &= *0F_1 + *0F_2 + u_4 \\
 x_5 &= *0F_1 + *0F_2 + u_5 \\
 x_6 &= *0F_1 + *0F_2 + u_6
 \end{aligned}
 \tag{14}$$

The zeroes indicate no influence of the factor on the indicator (in CFA language these parameters are “fixed” at zero). For instance, as can be seen in equations  $x_1$  and  $x_6$  (CFA) or the path diagram on the right in figure  $F_1$  influences  $x_1$  but not  $x_4$  and  $F_2$  influences  $x_6$  but not  $x_3$ .

This general pattern is an extreme example of what is referred to in EFA terms as simple structure. In EFA simple structure is a target of inductively oriented extraction and rotation algorithms, in CFA simple structure typically is assumed or imposed on the pattern of factor loadings.

## 5. LATENT CONSTRUCTS AND PROJECTED OR ACTUAL CORRELATIONS

In order to find latent construct(s), one must find an approximation  $\tilde{\Psi}$  of  $\Psi$ , the matrix of the specific variances, and then correct the correlation matrix of  $X$ , by  $\tilde{\Psi}$ . Using the matrix as a starting point, factor analysis examines the patterns of covariation represented by the correlation among items. Each item is then correlated with conceptual single latent construct – variable. The observed correlations between items can be recreated by appropriately multiplying the paths linking each pair of items via the factor.

To compute correlations efficiently between observed item responses and factor representing a latent construct(s), one can posit that the sum of all item responses is a reasonable numerical estimate of the one, all – encompassing latent construct that is assumed to account for interitem correlations. In essence, this overall sum is an estimate of the latent construct’s “score”. Because the actual scores for all items are presumed to be determined by one latent construct, a quantity combining information from all items (i.e. an overall sum) is a reasonable of that latent construct’s numerical value. With values those assigned to those causal pathways, one then can compute projected interitem correlations based on this one – factor model. These model – derived correlations are projections of what the actual interitem correlations should be if the premise of only one underlying variable is correct. The legitimacy of the premise can be assessed by comparing the projected correlations to the actual correlations. This amounts to subtracting each projected correlation from the corresponding actual correlation based on the original data. A substantial discrepancy between actual and projected correlations indicates that a single – factor model is not adequate, that there is still some unaccounted – for covariation among the items.

The residual matrix is supportive in latent constructs extraction and it can be treated in the same way as the original correlation matrix was treated, extracting e.g. a second factor and another corresponding to new latent construct(s). Once again, correlations between the items and that second latent construct can be computed and, based on those correlations, a matrix of proposed correlations can be generated. Those proposed correlations represent the extent of correlation that should remain among items after the second factor has been taken into account. If the second factor captured all of the covariation left over after extracting the first factor, then these projected values should be comparable to the values that were in the residual matrix. If not, further factors may be needed to account for the remaining covariation not yet ascribed to a factor. This process can proceed, with each successive factor being extracted from the residual matrix that resulted from the preceding iteration, until a matrix is achieved that contains only acceptably small residual correlations.

Also partial correlations can be explained when the effects of other items are taken into account. If “true” factors exist in the data, the partial correlations should be small, because the items can be explained by the items loading on the factors. The one exception regarding high correlations as indicative of a poor correlation matrix occurs when two items are highly correlated and have substantially higher loadings than other items on that factor. Then, their partial correlation may be high because they are not explained to any great extent by the other items, but do explain each other.

6. EXAMPLE ON LATENT CONSTRUCTS IN FACTOR ANALYSIS AND APPLICATION TO MARKETING SEGMENTATION

In order to detect, test and describe customers attitudes towards their professed values there was conducted: Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA). Afterwards, the extracted and tested factors were described with Multidimensional Scaling (MDS) method.

As far as the set of various items pertaining to human (customers) values is concerned in EFA and CFA, they all have been derived from earlier study and survey of young people (members of academic youth – students in age between: 20-24) [20]. Initially 22 items across various fields of human values interests were developed and introduced in questionnaire. They were as follows: 1 – “Freedom”, 2 – “Independence”, 3 – “Success”, 4 – “Enjoying life”, 5 – “Pleasure”, 6 – “Family”, 7 – “Honesty”, 8 – “Helpfulness”, 9 – “Forgiveness”, 10 – “Peace”, 11 – “Justice”, 12 – “Beauty”, 13 – “Harmony”, 14 – “Politeness”, 15 – “Honouring parents”, 16 – “Obedience”, 17 – “Wealth”, 18 – “Tradition”, 19 – “Moderation”, 20 – “Humbleness”, 21 – “Accepting my place in life” and 22 – “Devotion”. All items were measured on 5 point Likert scale.

Author fulfilled all conditions for the correctness of the above undertaken analysis e.g.: 1) variables have index meaning, 2) the measurement of all variables was carried out on Likert scale, 3) sample exceeded the minimum of 100 respondents ( $n = 232$ ).

The extracted and confirmed factors in EFA and CFA, were later described by MDS method on selected variables such as: 1) gender, 2) four types of personality, 3) three typically performed activities by respondents (see table 2). These variables belonged to group of psychodemographics area that are based on AIOD approach – “Activities-Interests-Opinions–Demographics” [11].

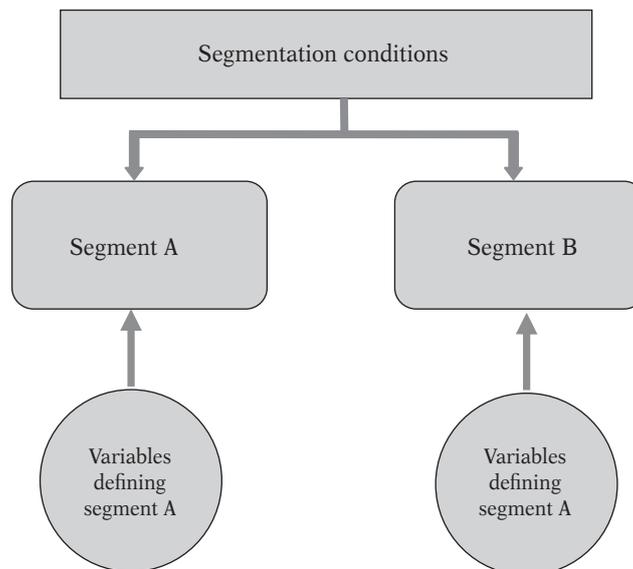


Figure 6. Relationship between segmentation conditions and its descriptors. Source: [11]

Thus one has described the relationship between descriptors and variables that constituted the principles of our segmentation [figure 6]. Besides in proceeding analysis there were only included those descriptors that might effectively differentiate segments. Descriptors that did not explain sufficiently differences in the process of segmentation of customers values were simply excluded from MDS.

Moreover there were used only values with highest factor loadings and all calculations were conducted in software: Statistica 9.0., SPSS 18.0 and Lisrel 8.0.

Table 2

Selected customers' variables from AIOD considered in segmentation analysis

Personality type	Performed activities	Gender
1. Technologically up-to-date: Aware of the possibilities that modern technology offers, and completely „on line” with the latest developments in technology	<ul style="list-style-type: none"> <li>• Party: Going out – partying and having fun</li> </ul>	Female
2. Extrovert: Friendly and outgoing; energized by being around other people; a person who is active and expressive	<ul style="list-style-type: none"> <li>• Watching TV</li> </ul>	Male
3. Advertising-conscious: Scan the print and electronic media for specials and bargains	<ul style="list-style-type: none"> <li>• Reading book</li> </ul>	
4. Locally oriented: Read local news and prefer to get it from newspapers rather than from television		
Measured on: Likert scale		Nominal scale

Source: [20].

### Stage I – Exploratory Factor Analysis (EFA)

Before analyzing structure of constructs, a scree plot with eigenvalues was initially examined, where one might have concluded there were at least 4 of them. On the observed plot of eigenvalues, what can be seen is an elbow that is visibly cracked and rapid. That's why in the first row we thought the sample should be extracted with only four constructs consisting of 22 items. The rest constructs on the shallow slope, contribute very little to our model. The last big drop actually occurs between the fourth and fifth component, so using the first four ones is an easy choice. There is also a visible one prevailing construct above the others. This construct (with number one) explains much part among all eigenvalues.

Also comparing a visual presentation (a scree plot) to extracted constructs by **method of principal components** (PC) and its variance being explained, it is right to say again, that there are at least 4 retained constructs with eigenvalues greater than 1. Those constructs with eigenvalues lower than 1, were eliminated from our further analysis. As we can observe (when 22 items were considered at first) the greatest distance

(table 3) appears between values  $\lambda_1 = 6,32$  and  $\lambda_4 = 1,15$  and later between  $\lambda_1 = 6,32$  and  $\lambda_3 = 1,55$  respectively. Distance created between 1 and 4 is the greatest.

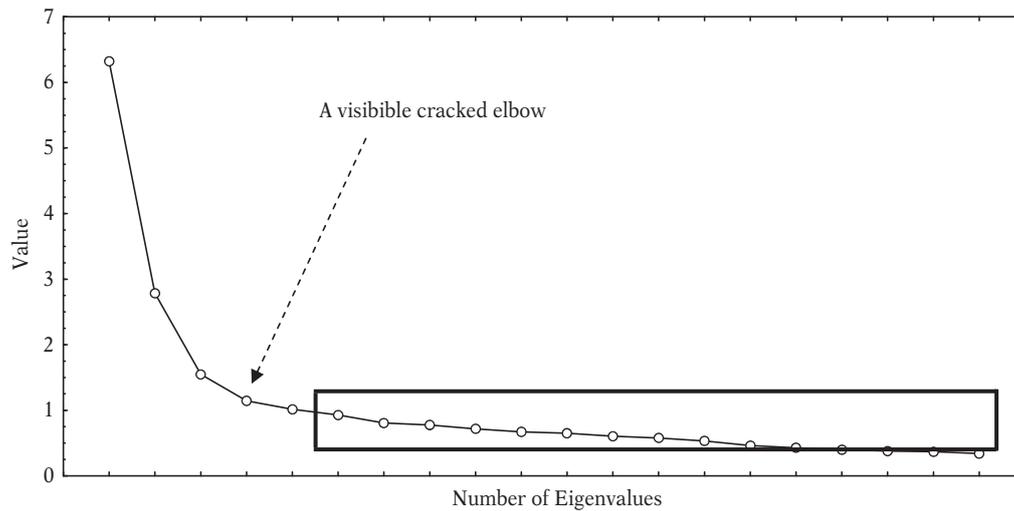


Figure 7. Plot of eigenvalues including all 22 item values

However after removing 8 items (observable variables), eigenvalues structure also changed. As a result the total variance level (for 4 constructs) was not accounted for (as before with 53,63%) but with 62,83% of variance.

Table 3

Eigenvalues extraction by principal components

	Eigenvalues at 22 items		
	Eigenvalue	% Total variance	Cum. % Total variance
$\lambda_1$	6,32	28,73	28,73
$\lambda_2$	2,79	12,66	41,39
$\lambda_3$	1,55	7,04	48,42
$\lambda_4$	1,15	5,21	<b>53,63</b>
Eigenvalues at 14 items – after reduction 8 other items			
$\lambda_1$	3,84	27,39	27,39
$\lambda_2$	2,45	17,49	44,88
$\lambda_3$	1,38	9,86	54,74
$\lambda_4$	1,13	8,08	62,83

Also the Kaiser's *KMO* criterion [7] enabled us to infer about higher level adequacy of above selected items for our model with 14 items. The index of *KMO* was:

$$KMO = \frac{\sum_p \sum_{h \neq p} r_{ph}^2}{\sum_p \sum_{h \neq p} r_{ph}^2 + \sum_p \sum_{h \neq p} \hat{r}_{ph}^2} = 0,74_{with\ 22\ items} \text{ and respectively } KMO = 0,79_{with\ 14\ items} \quad (15)$$

Here we can observe by the Kaiser's index, that it has increased when 8 items were excluded from further analysis. The level of *KMO*' interpretation pertaining to model quality assessment should be explained according to structure correlation matrix. They are: 0,9 – excellent, 0,8 – recommendable, 0,7 – decent, 0,6 – average, 0,5 – poor.

As a result we have focused only on 14 items (instead of 22) that were considered in further part of analysis (CFA).

In order to estimate correctly factor loadings in comparison with their items we've used four methods:

1. Principal Components (PC),
2. Maximum Likelihood (ML),
3. MINimum RESiduals (MINRES),
4. Generalized Least Squares (GLS).

As we can observe from table 4, factor loadings are slightly lower than in Principal Components method. Obtained values (when using ML, MINRES and GLS method) are lower than in PC. It is because factor analysis explains only communalities, not total variance. Hence the structure of final composite factors is changed and their interpretation must be also modified.

At last in the course of EFA (subjective analysis) we've identified initially four factors but in the next part of analysis CFA, two of them were correlated, which resulted in two final factors (table 7).

Table 4

Factor loadings structure after Varimax (PC, ML, MINRES) and Promax (GLS) rotation  
(with marked loadings that are >0,50)

Items	PC Principal Components method				ML Maximum Likelihood method			
	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$
1	,05	,18	,18	<b>,84</b>	,05	,27	<b>,54</b>	,20
2	,02	,22	,10	<b>,85</b>	,02	,23	<b>,97</b>	,08
3	,24	<b>,70</b>	-,07	,13	,20	<b>,53</b>	,13	,00
4	-,17	<b>,74</b>	,25	,18	-,11	<b>,74</b>	,14	,15
5	-,09	<b>,74</b>	,24	,17	-,03	<b>,72</b>	,14	,14

continued Table 4

Items	PC Principal Components method				ML Maximum Likelihood method			
	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$
6	,30	,01	<b>,74</b>	,12	,36	,03	,12	<b>,51</b>
7	,20	,08	<b>,81</b>	,22	,23	,10	,17	<b>,89</b>
8	<b>,64</b>	,23	,24	-,02	<b>,59</b>	,19	,00	,18
9	,19	<b>,64</b>	-,25	,04	,13	<b>,51</b>	,06	-,08
10	<b>,71</b>	,09	,00	,12	<b>,60</b>	,09	,03	,08
11	<b>,69</b>	,22	,19	-,17	<b>,67</b>	,18	-,06	,05
12	<b>,62</b>	-,02	,37	-,09	<b>,62</b>	-,01	,04	,17
13	<b>,74</b>	-,02	,11	,06	<b>,66</b>	,01	,07	,11
14	<b>,74</b>	-,16	,02	,12	<b>,64</b>	-,09	,03	,05
Items	MINRES method				GLS method (Promax rotation)			
1	,05	,24	<b>,61</b>	,20	,10	,37	<b>,61</b>	,36
2	,02	,24	<b>,85</b>	,09	,06	,40	<b>,93</b>	,29
3	,21	<b>,57</b>	,12	-,02	,24	<b>,57</b>	,26	,13
4	-,12	<b>,71</b>	,17	,18	-,04	<b>,77</b>	,33	,24
5	-,04	<b>,68</b>	,17	,17	,04	<b>,74</b>	,33	,25
6	,35	,03	,14	<b>,52</b>	,42	,12	,21	<b>,65</b>
7	,24	,09	,19	<b>,83</b>	,34	,24	,30	<b>,87</b>
8	<b>,59</b>	,19	,02	,17	<b>,62</b>	,23	,09	,35
9	,13	<b>,55</b>	,06	-,10	,15	<b>,52</b>	,15	,00
10	<b>,60</b>	,10	,06	,06	<b>,61</b>	,13	,08	,25
11	<b>,66</b>	,18	-,08	,09	<b>,69</b>	,19	,01	,25
12	<b>,61</b>	-,02	,00	,21	<b>,65</b>	,03	,08	,36
13	<b>,67</b>	,00	,05	,09	<b>,68</b>	,06	,10	,28
14	<b>,64</b>	-,10	,05	,04	<b>,63</b>	-,05	,04	,22

Configuration of factor loadings after rotation (14 items): 1 – “Freedom”, 2 – “Independence”, 3 – “Success”, 4 – “Enjoying life”, 5 – “Pleasure”, 6 – “Beauty”, 7 – “Harmony”, 8 – “Obedience”, 9 – “Wealth”, 10 – “Tradition”, 11 – “Moderation”, 12 – “Humbleness”, 13 – “Acceptance”, 14 – “Devotion”

## Stage II – Confirmatory Factor Analysis (CFA)

In this section we've implemented CFA to test the extracted factors. In CFA model we used:

1. Maximum Likelihood method (with Varimax rotation of Lawley's, where factor loadings proceeded through iterations). Thus we eliminated the uncertainty and subjectivity that strongly appeared in PC),
2. Generalized Least Squares (with Promax rotation) [1].

Applied **measures of goodness of fit** consisted of different types of indexes. For each of them results were displayed next to their mathematical notation. Their detailed description can be found in the work Szttemberg-Lewandowska [18].

The applied statistical test Chi-square (on the importance of fitting the model to data) in ML method (with function  $F_{ML}$ ) reached the level  $\chi^2 = 56,88$  at  $df = 41$  and  $p = 0,059$ . It points to a value equal 0,059 of significance and is therefore greater than commonly accepted 0,05. One must therefore reject the hypothesis of no linear relationship between observed variables (items). It is noteworthy that this level (of 0,05) is exceeded only 0,009 [table 5]. Therefore it can be further inferred that correlations between factors are either weak or they are strong only for sake of two factors (as evidenced by the values of the correlation factors in table 6).

The value of significance (in GLS method with the function  $F_{GLS}$ ) – reached a level of 0,152, and is much larger than 0,05.

Table 5

Goodness of fit measures for 4 factors

ML		
Chi-kwadrat	df	$\alpha$
56,88	41	0,059
GLS		
50,28	41	0,152

Calculated indexes **GFI (Goodness-of-Fit)** and **AGFI (Adjusted Goodness-of-Fit)** by Joreskog and Sorbom) – (with the range [0, 1]) mark a good match to presented data model:

$$GFI = 1 - \frac{F[S, \Sigma(\hat{\theta})]}{F[S, \Sigma(\theta)]} = 0,93 \text{ and} \quad (16)$$

$$GFI = 1 - \left(\frac{c}{df_h}\right) \frac{F[S, \Sigma(\hat{\theta})]}{F[S, \Sigma(\theta)]} = 1 - \frac{c}{df_h}(1 - GFI) = 0,90$$

The high level of fit in the data is also proved by indexes such as **NFI (Normed Fit)** by Bentler and Bonett, – whose values typically are placed in range of [0, 1]

and *NNFI* – **Nonnormed Fit** Bentler and Bonett – where values fall even outside the range [0, 1]):

$$NFI = \frac{\chi_i^2 - \chi_h^2}{\chi_i^2} = \frac{F_i - F_h}{F_i} = 1 - \frac{F_h}{F_i} = 0,88 \quad (17)$$

$$NNFI = \frac{(\chi_i^2 / df_i) - (\chi_h^2 / df_h)}{\chi_i^2 / df_i - 1} = \frac{(F_i / df_i) - (F_h / df_h)}{(F_i / df_i) - (1 / (n - 1))} = 0,93 \quad (18)$$

**Parsimony Goodness-of-Fit** (by James, Mulaik and Brett) gives penalty for less restricted models (where more estimated parameters appear). In this index, values are always smaller than by indexes *GFI* and *AGFI*.

$$PGFI = \frac{df_h}{df_n} GFI = 0,69 \quad (19)$$

And finally the calculated **RMSEA (Root Mean Square Error of Approximation)** reached the level of 0,06. It measured the bad fit of data associated with model. Theoretically the closer its value to zero, the better the level of fit.

From table 6, we notice actually no correlations between other factors (which in fact proves to be orthogonal) except: 1) “Hedonistic Consumerism” and “Freedom – Independence” and 2) “Life Sensitiveness” and “Conservatism”. Therefore it was decided to merge factors with numbers: 1 and 3 and respectively 2 with 4 (table 7).

Table 6

Correlations between factors in ascending order

Factors	Level
Free Independence – Conservatism	0,13
Life Sensitiveness – Hedonistic Consumerism	0,10
Hedonistic Consumerism – Conservatism	0,11
Life Sensitiveness – Free Independence	0,12
Hedonistic Consumerism – Free Independence	<b>0,52</b>
Life Sensitiveness – Conservatism	<b>0,54</b>

Stage III – Interpretation of constructs in context of their utility for marketing activities

Initially in **EFA** model, we found four constructs altogether (table 7). Their contents were interpreted against the background of different areas aggregating particular

items. Construct one consisted of items such as: “Obedience”, “Tradition”, “Moderation”, “Humbleness”, “Acceptance”, and “Devotion”. Owing to specific structure of this composition and items characteristics, we’ve decided to give it a name “**Conservatism**”. But after **CFA** “Conservatism” construct was supplemented with two more items taken out from construct “Life sensitiveness”. Those added items were “Beauty” and “Harmony”. Respondents in this group highly valued attributes of nature and arts, serenity, peace, predictability and stabilization in every field of their live.

Table 7

Factors extracted in EFA and confirmed in CFA

Exploratory Factor Analysis			
Conservatism (1)	Hedonistic Consumerism (2)	Life Sensitiveness (3)	Freedom – Independence (4)
Obedience	Success	Beauty	Freedom
Tradition	Enjoying life	Harmony	Independence
Moderation	Pleasure		
Humbleness	Wealth		
Acceptance			
Devotion			
Confirmatory Factor Analysis			
Conservatism + Life Sensitiveness	Hedonistic Consumerism + Freedom – Independence		
Obedience	Success		
Tradition	Enjoying life		
Moderation	Pleasure		
Humbleness	Wealth		
Acceptance	<u>Freedom</u>		
Devotion	<u>Independence</u>		
Beauty			
Harmony			

Legend: Items underlined, mean they have been added on CFA model

Next construct – “**Hedonistic Consumerism**” was loaded with items such as: “Success”, “Enjoying life”, “Pleasure”, “Wealth”. This type of construct was also supplemented with items such as “Freedom”, “Independence”.

Stage IV – Multidimensional scaling (MDS) segmentation on constructs and AIOD variables

In MDS analysis, confirmed factors were analyzed across selected AIOD variables (table 2). The principles of MDS can be summarized in three-stage process:

- examination of structure containing objects or variables in matrix by defining their dimensions on the basis of similarities or preferences,
- visualization of data in  $n$  – dimensional space at  $r < m$  relationships taking place between analyzed objects or variables,
- calculation distances  $\hat{d}_{ij}$  in matrix and finding accurate level of data fit in the model.

In our example, we chose ALSCAL model (*Alternating Least Squares Scaling*) based on iterations. In this model one can take into account the measurement based on: 1) scale – both interval, ratio or order, 2) complete and incomplete data, 3) symmetrical and asymmetrical designated values of the data [12]:

$$S \left\{ \frac{1}{m} \sum_{k=1}^m \left[ \frac{\sum_i \sum_j (d_{ij,k}^2 - \hat{d}_{ij,k}^2)^2}{\sum_i \sum_j \hat{d}_{ij,k}^4} \right] \right\}^{\frac{1}{2}}. \quad (20)$$

ALSCAL is preceded by SSTRESS function [19]:

$$SS = \sum_i \sum_j \sum_k (d_{ij,k}^2 - \hat{d}_{ij,k}^2)^2. \quad (21)$$

More detailed information about the proces of model estimation, can be found in Zaborski's work [23].

In order to check the level of fit in our data with considered model we used additionally the squared correlation  $RSQ$  [12] which explains the proportion of variance of the scaled data in the partition that is accounted by its corresponding distance. In  $RSQ$  it is assumed that when using the method of MDS, an  $S$  – value less than 0,2 is “considered acceptable” for 2-dimensional solution. As a result  $RSQ$  square is high enough to capture substantial proportion of variance if it is above 85%. If two conditions are met (e.g.  $S < 0,20$  and  $RSQ > 0,85$  a 2-dimensional solution is a parsimonious one, that provides a sufficient explanatory power. A 3-dimensional one is “more complex” and adds little explanatory power. Thus a 2-dimensional solution (as it is in our case both for Male and Female group) can sufficiently reflect the information embedded in data.

Normally, ALSCAL is used to provide a visual representation of a complex set of relationships that can be scanned at a glance. Since maps on paper are 2-dimensional objects, this method translates technically to finding an optimal configuration of points in 2-dimensional space. However, the best possible configuration in 2 dimensions may be a very poor, highly distorted, representation of data. If so, this will be reflected in a high stress value. When this happens, one has two choices: one can either abandon

ALSCAL as a method of representing of data, or one can increase the number of dimensions. The problem is that with increasing dimensions, researcher must estimate an increasing number of parameters to obtain a decreasing improvement in stress. The result is model of the data that is nearly as complex as the data itself.

In our MDS analysis, the **male** group of respondents achieved  $S = 0,163$  and  $RSQ = 0,887$  in fourth iteration and respectively in **female** group  $S = 182$  and  $RSQ = 0,858$  in sixth iteration. Therefore we can infer that both models fit the data suitably. Obviously when the MDS map perfectly reproduces the input data,  $f(x_{ij}) - d_{ij}$  that is for all  $i$  and  $j$ , the stress is then zero. Thus, the smaller the stress, the better the representation.

Table 8

Coordinates for items (observed variables) and selected AIOD variables

Items	Male		Female	
	Dim. 1	Dim. 2	Dim. 1	Dim. 2
1 – „Obedience”	0,54	-0,20	-0,53	-0,01
2 – „Tradition”	0,68	0,20	-0,86	0,15
3 – „Moderation”	0,62	-0,37	-0,84	-0,31
4 – „Humbleness”	0,96	-0,27	-0,85	0,45
5 – „Acceptance”	0,88	-0,24	-0,98	0,26
6 – „Devotion”	1,05	0,05	-1,12	0,02
7 – „Success”	-0,69	-0,17	-0,37	-0,56
8 – „Enjoyment”	-0,97	-0,45	0,33	-0,59
9 – „Pleasure”	-1,04	-0,31	-0,26	-0,44
10 – „Wealth”	-0,97	0,05	-0,54	-0,99
11 – „Beauty”	0,59	-0,60	-0,43	0,42
12 – „Harmony”	-0,03	-0,32	-0,41	0,70
13 – „Freedom”	-0,61	0,12	-0,01	0,53
14 – „Independence”	-0,70	-0,03	0,14	0,35
15 – „Technology”	0,50	1,36	0,93	0,39
16 – „Extrovert”	-0,49	1,14	0,96	0,79
17 – „Advert”	0,70	1,03	0,74	1,07
18 – „Local”	-1,03	0,81	1,11	-0,11
19 – „Party”	0,08	0,85	1,64	-0,05
20 – „Tv”	0,21	-1,22	0,02	-1,01
21 – „Book”	-0,25	-1,43	1,32	-1,05

Legend:

**Item Values:** 1 – “Obedience”; 2 – “Tradition”; 3 – “Moderation”; 4 – “Humbleness”; 5 – “Acceptance”; 6 – “Devotion”; 7 – “Success”; 8 – “Enjoyment”; 9 – “Pleasure”; 10 – “Wealth”; 11 – “Beauty”; 12 – “Harmony”; 13 – “Freedom”; 14 – “Independence”.

**Personality Types:** 15 – “Technology” (Technologically up-to-date); 16 – “Extrovert”; 17 – “Advert” (Advertising-conscious), 18 – “Local” (Locally oriented)

**Performed activities:** 19 – “Party” (going out for a party); 20 – “Tv” (watching Tv); 21 – “Book” (reading book)

When looking at a map that has non-zero stress, one must keep in mind that the distances among items are imperfect and representations of the relationships given by data are distorted. The greater the stress, the greater the distortion. However, one can rely on the larger distances as being also accurate. This is because the stress function sometimes accentuates discrepancies in the larger distances. Fortunately in our case, – the analyzed data and calculated Stress values for both groups reflect good fit.

Having based on 2 maps (with variables – see tables 2 and 7) one can make their appropriate interpretation. There is still one (technically speaking) important thing to realize about those maps. That is the orientation of the picture is arbitrary. Thus map representation of distances between customers' values and their personality characteristics as well their selected activities, must not be oriented such that "**Conservatism**" (with set of items) is up and "**Hedonistic Consumerism**" (set of items) is down. In fact, "Conservatism" might be down and "Hedonistic Consumerism" might be up. All that matters in this type of interpretation with these maps is which point is close to others and where "reasonable dimensions" or at least coherent configurations exist.

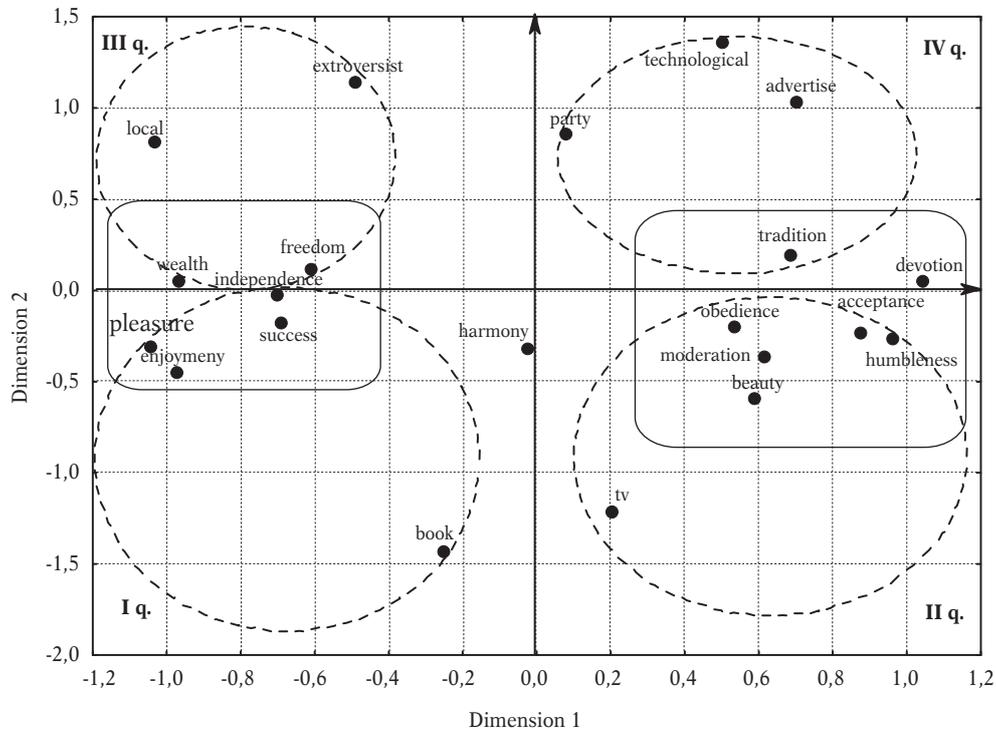
#### Stage V – Interpretation of MDS results in marketing context

In the map 1 and the dimension 1 (describing young men from **male group**), we may conclude that the those respondents valuing "Pleasure", "Enjoyment", "Success", "Freedom", "Wealth", "Independence" from one hand, are inherently "Extroverts" and "Locally oriented" and spend their spare time on reading books. Members of this group are generally well-read, hard – headed, goal-oriented and intelligent.

In turn, young people – e.g. in group of men who prefer "Tradition", "Acceptance", "Humbleness", "Moderation" and "Devotion" are more often "Advertising-conscious" and even "Technologically up-to-date", which makes them more susceptible to advertising campaigns and all kinds of technological product innovations such as new types of mobile phones. They prefer to spend their time on watching television or entertainment activities outside home.

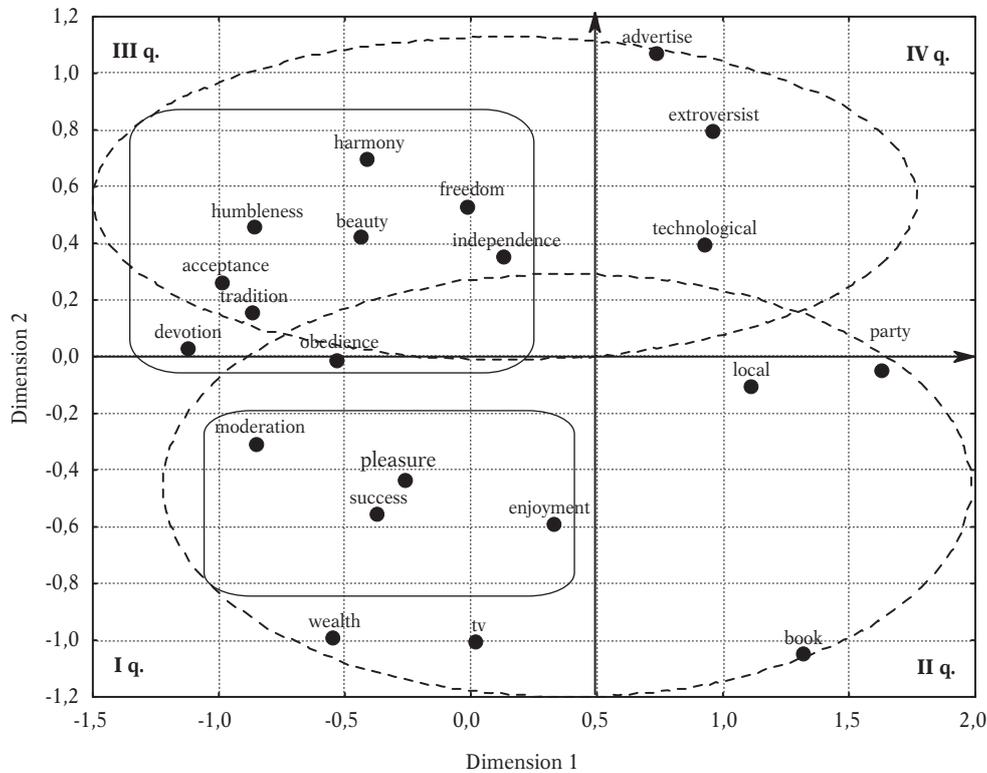
Men distinguish from young women by their professed values and their personality types as well their activities.

Moreover as we can observe (map 1) in first quarter, values such as: "Success", "Enjoyment", "Pleasure", or "Independence" are related to activity "Book reading". In second quarter there are values: "Obedience", "Acceptance", "Moderation", "Humbleness" and quite surprisingly value – "Beauty" that is connected with "Tv watching". In third quarter, "Independence", "Wealth" and "Freedom" is related to "Locally oriented" youth and "Extroverts".



Map 1. Male group of respondents

As far as **female group** of respondents is concerned (2 map), distinctions are not so clear as compared to male group. Young women seem to be more of complex character and personality type. Simply saying they have contrary configuration of traits as compared to men's group. First of all, they highly value: "Devotion", "Humbleness", "Tradition", "Acceptance", and "Harmony", but at the same time they're strongly interested in preserving in their life values such as: "Freedom", "Independence" or sustaining "Beauty". Obviously they strive to hold on to and mix "Conservatism" with "Hedonistic Consumerism" values. And those women (in the first quarter) who prefer "Pleasure", "Success" and "Enjoyment", also long for "Moderation", which might be a little bit confusing in men's map configuration.



Map 2. Female group of respondents

Actually values and AIOD variables configurations in women group, are mixing each other and they are not so constant as seen earlier in men's group. It also varies across the choice of selection within direction of analysis that is depending on particular dimension description. In short it changes from the perspective of visual interpretation by researcher. However it must be admitted that, women (in third and fourth quarters) are "Extroverts", "Technologically up-to-date", "Advertisement-conscious" and simultaneously hold on to "Conservatism" and "Hedonistic Consumerism" (jointly considered – as a certain way in their life style). Also young women in the first and second quarters are more focused on "Hedonistic Consumerism" and they are rather: "Locally oriented", "Book reading" and "Tv watching".

## 7. CONCLUSIONS

Finding the exact measurement model and method of estimation of the scores in constructs (otherwise called latent variables or latent constructs) can be extremely difficult challenge for any researcher. It is because of many various alternatives that

analyst must simultaneously consider and choose, according to which a final (appropriate or misleading) composition of constructs will be made. Certainly the measurement statistical model helps to some extent understand and evaluate the scores coming from the item responses making up a real construct. However by the time this construct will be generated, one must along the way overcome dilemmas about which type of model and method of estimation to choose, to make the best of it (e.g. either model focusing only on the items and their relationship to the construct or model focusing on the scores and their relationship to the construct).

Analytical problems also arise on the course of translation from scored responses to locations on the respondents' construct map – (which are made up of collections of items grouped into a composite score, not readily observable by direct means). Problems are usually related to:

- the distance between respondent and response on the construct map,
- distance between different responses on the construct map,
- difference between different respondents.

And number of  $p$  items (held in the data matrix) with quite various characteristics unfortunately don't simplify the identification of projected constructs and their later explanation. For instance using factor analysis, it is actually possible to locate latent constructs, but on the other hand it is sometimes difficult to judge whether selected for analysis items and grouped together by extracted construct are the right ones. It is because items can be (or should be) characterized not only on communality coefficients but also in relation to the interpretability of particular factors (constructs), in short, the items' played role (sense or meaning) in the interpretation of construct. Here once again the matter is open, whether choice of subjective or objective methods in factors (constructs) consideration would be a better solution. Usually these aspects are open and typically they are choices to be made.

As far as two constructs are concerned (confirmed in our analysis) one can assuredly make the following conclusions for marketing purposes. "Conservatism" and "Hedonistic Consumerism" constructs can be characterized towards certain targeted by companies segments of the market. Each construct represents not only a set of items, but above all it provides marketing information about suitable segment and its customers (with applications not only to youth segment but also to many others). Simply saying those constructs can work to marketing's advantages. Through them one may generate real ideas – concepts of marketing strategy and undertake effective actions presence of different segments in the area of: 1) promotion (e.g. expression of catching advertising slogans or selection of media communication such as: TV, press, radio or Internet); 2) choice of appropriate distribution channels based on indirect or direct customer preferred contact; 3) improved attributes in products and methods of product launch into the market or 4) setting the adjusted price level for products according to customers' personal preferences.

## REFERENCES

- [1] Balicki A., [2009], *Statystyczna analiza wielowymiarowa i jej zastosowania społeczno-ekonomiczne*, UG, Gdańsk.
- [2] Bartholomew D.J., Steele F., Moustaki I., Galbraith J.I., [2008], *Analysis of multivariate social science data*, 2<sup>nd</sup> ed. Chapman and Hall Books, New York.
- [3] Bentler P.M., Weeks D.G., [1980], *Linear structural equations with latent*, [in:] Long, J.S. (ed.), *Testing Structural Equation Models*, Sage Publications, New York.
- [4] Bąk A., [2007], *Application of latent variable models to consumers preferences analysis*, „Acta Universitatis Lodzianis: Folia Oeconomica”, Uniwersytet Łódzki, pp. 321-329.
- [5] Guttman L., [1945], *A basis for analyzing test-retest reliability*, „Psychometrika”, 10, pp. 255-282.
- [6] Guttman L., [1954], *Some necessary conditions for common-factor analysis*, „Psychometrika”, 19, pp. 149-161.
- [7] Kaiser H.F., [1960], *The varimax criterion for analytic rotation in factor analysis*, „Psychometrika” 23, pp. 187-200.
- [8] Kruskal J.B., [1964], *Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis*, „Psychometrika”, 29, pp. 1-28.
- [9] Loehlin J.C., [2004], *Latent variable models: an introduction to factor, path, and structural equation*, Lawrence Erlbaum Associates, New Jersey.
- [10] Marcoulides G.A., Moustaki I., [2002], *Latent variable and latent structure models*, Lawrence Erlbaum Associates, New York.
- [11] Sagan A., [2004], *Badania marketingowe – podstawowe kierunki*, UE Kraków.
- [12] Schiffman S.S., Reynolds M.L., Young F.W., [1981], *Introduction to multidimensional scaling: the theory, methods and applications*, Academic Press, New York.
- [13] Skrondal A., Rabe-Hesketh S., [2004], *Generalized latent variable modeling*, Chapman and Hall, New York.
- [14] Spearman C., [1904a], *The proof and measurement of association between two things*, „American Journal of Psychology”, 15, pp. 72-101.
- [15] Spearman C., [1904b], *General intelligence objectively determined and measured*, „American Journal of Psychology”, 15, pp. 201-293.
- [16] Spearman C., [1910], *Correlation calculated from faulty data*, „British Journal of Psychology”, 3, pp. 271-295.
- [17] Stevens S.S., [1946], *On the theory of scales of measurement*, „Science”, 103, pp. 667-680.
- [18] Szttemberg-Lewandowska M., [2008], *Analiza czynnikowa w badaniach marketingowych*, UE Wrocław.
- [19] Takane Y., Young F.W., de Leeuw J., [1978], *Nonmetric individual differences in multidimensional scaling: an alternating least squares method with optimal scaling features*, „Psychometrika”, 42, pp. 7-67.
- [20] Tarka P., [2008], *From ranking (Rokeach – RVS) to rating scales evaluation – some empirical observations on multidimensional scaling Polish and Dutch youth’s values*, „Innovative Management Journal”, 1, 2, pp. 24-42.
- [21] Tarkkonen L., [1987], *On reliability of composite scales*, Statistical studies 7 ed. Finnish Statistical Society.
- [22] Thurstone L.L., [1947], *Multiple factor analysis*, The University Press, Chicago.
- [23] Zaborski A., [2001], *Skalowanie wielowymiarowe w badaniach marketingowych*, AE Wrocław.
- [24] Wilson M., [2005], *Constructing Measures: An Item Response Modeling Approach*, Lawrence Erlbaum Associates, New York.

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MODELE ZMIENNYCH UKRYTYCH – ROZWAŻANIA NAD POMIAREM I GENEROWANIEM  
KONSTRUKTÓW W SFERZE WARTOŚCI KONSUMENTÓW

## Streszczenie

W artykule autor wymienia i opisuje różne typy modeli pomiarowych w zakresie tzw. zmiennych ukrytych. W pracy rozważa on także metodę i technikę analizy wyników w ramach identyfikacji ww. zmiennych (w literaturze występujących m.in. pod nazwą „konstruktów”). W niniejszej pracy omawiane są również zagadnienia związane z identyfikacją tego typu zmiennych z perspektywy poziomu pomiaru tj.: 1) odległości pomiędzy respondentami i ich odpowiedziami na tzw. mapie konstruktów (zmiennej); 2) odległości pomiędzy samymi odpowiedziami na mapie i 3) badaniu różnic pomiędzy respondentami. Dalsza część artykułu skoncentrowana jest na opisie dwóch klasycznych modeli analizy zmiennych ukrytych, opartych m.in. na metrycznych cechach pomiarowych: EFA (Eksploracyjnej Analizie Czynnikiemowej) i CFA (Konfirmacyjnej Analizie Czynnikiemowej). W pierwszej kolejności rozważana jest eksploracyjna analiza czynnikowa, którą to autor wykorzystuje do identyfikacji i opisu konstruktów (wartości konsumentów). W wyniku tej aplikacji (na podstawie 22 włączonych do analizy pozycji i ich wyników zgromadzonych z wcześniejszych przeprowadzonych badań empirycznych), wygenerowano wstępnie 4 konstrukty, którym nadano następujące nazwy: „Konserwatyzm”, „Wolność”, „Hedonistyczny konsumeryzm” i „Wrażliwość życiowa” – inaczej „Wrażliwość na otoczenie”. Następnie autor tworzy modeli CFA, gdzie redukuje liczbę zmiennych z 22 do 14 i ostatecznie tworzy 2 konstrukty: „Konserwatyzm” i „Hedonizm konsumpcyjny”. Na końcu artykułu, wyodrębnione konstrukty opisano na wybranych zmiennych z modelu AIOD, stosując do tego celu skalowanie wielowymiarowe. Konstrukty omówiono również w kontekście działalności marketingowej przedsiębiorstw.

**Słowa kluczowe:** Ukryte modele, konstrukty, wartości konsumentów

LATENT VARIABLE MODELS – ISSUES ON MEASUREMENT AND FINDING EXACT CONSTRUCTS  
IN CUSTOMERS' VALUES

## Summary

In article author defines different measurement latent models and describes specify of measurement latent constructs. In literature some examples of these models are: the “true – score” model of classical test theory, the “domain score” model, item response model, factor analysis and latent class models. This work also presents method of estimation that should be undertaken in the identification process of latent constructs. Some aspects related with adjustments in the measurement model depending on distance between respondent and their responses, are also discussed. Author describes them from the prospect of 1) distance between respondent and response on the construct (variable) map; 2) distance between different responses on the construct map and 3) difference between different respondents. Next going on to further description, author considers two types of models based on metrical items characteristics: EFA and CFA. In the exploratory factor analysis as a key latent variable model in constructs detection and their formulation is defined where 4 latent constructs are extracted. These four detected constructs (based on earlier set of 22 value items) were given the following names: “Conservatism”, “Freedom-Independence”, “Hedonistic Consumerism”, and “Life Sensitiveness”. Secondly there is implemented CFA model which reduces number of value items from 22 to 14, containing only two latent constructs called “Conservatism” and “Hedonistic Consumerism”. Additionally those constructs were in the end, described with selected AIOD variables where MDS was applied. And at last constructs were defined in context of their utility for marketing activity.

**Key words:** Latent models, constructs, customers' values